Understanding Momentum and Reversal*

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August 16, 2019

Abstract

Stock momentum, long-term reversal, and other price trend characteristics predict future realized betas. This helps explain why these characteristics predict returns—because they capture time-varying risk compensation. We formalize this argument with a conditional factor pricing model. Using instrumented principal components analysis, we estimate latent factors with time-varying factor loadings that depend on observable firm characteristics. Momentum and long-term reversal alphas are small and insignificant once we account for common risk factors in this way.

Keywords: momentum, reversal, factor model, conditional betas, conditional expected returns, IPCA

*We thank seminar participants at ASU, Duke, and Vanderbilt for helpful comments. AQR Capital Management is a global investment management firm, which may or may not apply similar investment techniques or methods of analysis as described herein. The views expressed here are those of the authors and not necessarily those of AQR.

1 Introduction

Since its introduction by Jegadeesh and Titman (1993) 25 years ago, the momentum anomaly has consistently ranked among the most thoroughly researched topics in financial economics. It forms the basis of strategies implemented throughout the asset management industry, and underlies a wide range of mutual funds and exchange traded products. Despite its widespread influence on the finance profession, momentum remains a mysterious phenomenon. A variety of positive theories, both behavioral and rational, have been proposed to explain momentum, but none are widely accepted.[1] It also remains one of the few reliable violators of prevailing empirical asset pricing models such as the Fama and French (2015) five-factor model. Indeed, research has generally failed to identify a risk exposure that can explain the consistent differences in assets' expected returns associated with recent price performance.

The objective of this paper is to re-evaluate the momentum anomaly through the lens of *conditional* asset pricing models. Our central finding is that the bulk of the momentum effect is explained by a conditional model. That is, by constructing factors that more accurately represent the conditional risk-return tradeoff, and with a proper specification of the time-varying betas on these factors, we find that momentum returns reflect compensation for exposure to aggregate risks. In short, momentum profits are explainable with betas alone and no anomaly intercept.

Our analysis begins from the simple observation that if expected returns vary persistently over time, then one will generically detect a momentum effect. Let $r_{i,t+1}$ denote the return of asset i in excess of the risk-free rate. It is comprised of its conditional expectation ($\mu_{i,t} \equiv E_t[r_{i,t+1}]$) and an unforecastable shock:

$$r_{i,t+1} = \mu_{i,t} + \epsilon_{i,t+1}. \tag{1}$$

The momentum strategy calculates recent moving averages of returns on each asset and forms long/short positions based on these. Momentum is, in essence, a rudimentary yet generally-applicable statistical filter for estimating assets' latent conditional means. It is rudimentary because the filtering algorithm—an equal-weighted moving average with fixed window size—is ad hoc and suboptimal.[2] It is generally applicable because it requires no

[1] Examples include Hong and Stein (1999), Vayanos and Woolley (2013), and Liu and Zhang (2014), among others.

[2] For example, the Kalman filter (under the additional assumption of a linear time series model for $\mu_{i,t}$) would estimate the $\mu_{i,t}$ process as an exponentially-weighted moving average of past returns, assigning higher weight to more recent returns but using all available historical data rather than a fixed window. Only under very strict and implausible assumptions would the standard momentum treatment constitute an optimal

knowledge or assumptions of parametric structure, just an educated guess that $\mu_{i,t}$ is serially correlated. The logic of the filter is transparent: Assets with the highest recent returns tend to be those with high $\mu_{i,t}$, and these are the same assets that will tend to have high future expected returns if $\mu_{i,t}$ is persistence (and vice versa for assets with low recent returns). Momentum, when viewed this way, is a clever and robust tactic for capturing persistent time-varying expected returns while requiring minimal understanding of the structure in returns.

The view that momentum strategies serve as a "crude filter" for expected return dynamics similarly applies to long-term reversal with small modification. If, for example, $\mu_{i,t}$ has strong positive AR(1) dynamics, $\mu_{i,t}$ will exhibit positive serial correlation at short and intermediate lags in essentially any realized sample (giving rise to a momentum effect). However, the same persistence can give rise to misleading statistical conclusions in terms of long-term return predictability. In fact, strong persistence in expected returns introduces a high propensity for (falsely) detecting significant negative serial correlation at long lags (i.e., long-term reversal).[3] That is, the same dynamics that give rise to momentum often lead to *false* detection of significant long-term reversals.

If momentum and long-term reversal generically arise from dynamic expected returns, what makes them "anomalous" phenomena? The decomposition in (1) is not a model, it is tautologically true. It applies equally well when $\mu_{i,t}$ is driven by arbitrage-free compensation for systematic risk exposure as it does when $\mu_{i,t}$ is dominated by arbitragable mispricings. Our motivating question seeks to distinguish between these two. Can we identify a model for $\mu_{i,t}$ that both satisfies the zero alpha condition of a no-arbitrage risk-return tradeoff, while at the same time matching the impressive quantitative magnitudes of price trend effects in the data? That is, can we identify a model in which momentum or reversal effects represent compensation for exposure to risk?

We first establish a critical necessary condition for the arguments outlined above. We show that momentum is a strong forecaster of future realized betas on common risk factors (such as those in the Fama-French three-factor model). In panel regression, we find that the momentum characteristic has highly significant predictive power for realized market betas over the subsequent year. Long-term reversal has similar predictive power for betas (short-term reversal is substantially weaker). The estimated predictive coefficient indicates that

filter.

[3]In Appendix A1, we show via simulation how short/intermediate term positive serial correlation and long-term negative serial correlation coexist in a simple system that has strictly positive AR(1) dynamics. The simulation model that we analyze is closely linked to the empirical properties of expected returns in our sample.

when a stock moves from the 10^{th} percentile of momentum returns to the 90^{th} percentile, its market beta increases by 0.15. This is a preliminary indication that momentum can be viewed, at least in part, as a risk-based return phenomenon.

To rigorously investigate whether this beta predictability evidence *quantitatively* rationalizes the average return patterns associated with price trends, we require an asset pricing model. To this end, we analyze a conditional factor pricing model of the form

$$r_{i,t+1} = \beta_{i,t}' \lambda_t + \epsilon_{i,t+1}. \tag{2}$$

In this framework, conditional expected returns $\mu_{i,t}$ are restricted to derive only from exposures ($\beta_{i,t}$) to a set of common risk factors and the associated factor premia (λ_t). At a minimum, a successful model will need to explain three facts, 1) a large spread in average returns of around 9% per annum for stocks in the highest quintile of past one year returns over those in the lowest quintile, 2) that a 12-month moving average produces better return predictions than alternative moving average windows, and 3) the marginally significant long-term reversal pattern.

Consider a static version of (2), so that $\mu_{i,t} = \beta_i' \lambda$ for all t. One condition alone—a sufficiently large spread in β_i—would match momentum's large average return spread. But it would also imply that a very long moving average window would outperform the standard 12-month momentum look-back, in contradiction of the well known pattern in the data. In other words, the traditional 12-month momentum implementation prefers to rapidly turnover stock constituents in the high and low deciles. The empirical probability of a stock transitioning out of the extreme quintiles is roughly 38% per month. This is not simply noise—longer moving average windows would mechanically reduce turnover, but this also reduces forecasting power.

The implication is that the identities of stocks with the highest and lowest conditional expected returns are changing over time. For a factor model like (2) to match the data, it needs to be a *conditional* model. Holding the factor premia fixed, dynamic betas ($\beta_{i,t}$) induce variation in the panel of $\mu_{i,t}$'s. This will produce churn in the list of stocks at the top and bottom of the $\mu_{i,t}$ distribution, particularly when there are multiple factors. The dynamics of factor risk premia (λ_t), layered on top of beta variation, could further magnify churn in stock-level conditional expected returns.

While conditional factor models offer a potential conceptual explanation for momentum and other price trend patterns, they pose a difficult estimation challenge. One estimation option is to use observable factors and estimate rolling betas. But observable factors may be misspecified—especially if reinterpreted as conditional factors when they were originally constructed for use as unconditional factors (e.g. Fama-French factors). And rolling betas may suffer a "staleness bias" as they only slowly incorporate conditioning information. Another option is to estimate monthly realized betas from daily data. But with only 20 observations per month, realized betas tend to be noisy, and they do not resolve the problem of misspecification in observable factors.

Instead, we follow the conditional factor modeling approach of Kelly, Pruitt and Su (KPS, 2019). They use the method of instrumented principal components analysis (IPCA), which estimates latent factors and factor exposures by parameterizing $\beta_{i,t}$ as a function of observable asset characteristics. By conditioning betas on observable time-varying characteristics, the model can quickly update risk exposures based on characteristic news. And, by estimating factors that best associate with conditional risk exposures, IPCA is freed from using prespecified factors that are prone to misspecification. KPS demonstrate that conditional factor models estimated via IPCA offer highly significant improvements in describing the cross section of risk and return compared to leading alternatives (such as Fama-French factors with rolling betas).

Our central empirical finding is that estimates of $\mu_{i,t} = \beta_{i,t}'\lambda$ from our conditional factor model offer economically large and statistically significant return forecasting improvements over the traditional momentum effect. Moreover, stock momentum and long-term reversal become statistically insignificant once we control for model-based $\beta_{i,t}'\lambda$. The statistical conclusion is that a more careful filter—one that imposes economic structure via a factor-based risk-return tradeoff and incorporates conditioning variables—is significantly more efficient at eliciting $\beta_{i,t}'\lambda$ than an ad-hoc moving average filter. More importantly, the economic conclusion is that momentum should *not* be viewed as an anomaly per se. Because the conditional factor model imposes the no-arbitrage zero-alpha condition, cross-sectional differences in model-based $\beta_{i,t}'\lambda$ arise solely from differences in assets' conditional covariances with aggregate factors. Therefore, we conclude that the cross-sectional momentum strategy achieves significant average returns because it sorts stocks on conditional risk exposures, and not because it sorts on temporary mispricings, as has often been suggested.

Our research question is most closely related to three precursors in the momentum literature. First, Conrad and Kaul (1998) suggest that differences in stock's expected returns explain momentum returns. Jegadeesh and Titman (2002) argued against this interpretation because it is based on unconditional expectations that they show are not dispersed enough to explain the momentum returns. Furthermore, as mentioned above, an explanation based on *unconditional* expectations will not generate the churn in the list of stocks involved in the momentum strategy. On the other hand, an explanation based on *conditional* expectations does.

Second, Grundy and Martin (2001) decompose returns into a systematic risk component (captured as exposure to the three Fama-French (1993) factors) and stock-specific residuals, and find that the momentum phenomenon is driven entirely by momentum in residual returns. We find that the Grundy and Martin (2001) conclusion is driven by factor model misspecification—in their case due to rolling-window betas on observable factors. Using a model with slow-moving betas and erroneous factors all but ensures that residuals inherit the important variation in expected returns, giving the misleading impression that momentum is a feature of idiosyncratic returns. Our conclusions are exactly the opposite. Through a careful specification of factors and conditional beta dynamics, we offer an improved factor model that overturns earlier findings.

Third, Chordia and Shivakumar (2002) decompose stock returns into a component that is forecastable with macroeconomic predictor variables and an unforecastable shock. They conclude that momentum returns are best captured through the conditional expected returns predicted by macroeconomic variables, rather than through the residual. They conjecture that the predictable component proxies for dynamic factor-risk premia (in contrast to the conclusions of Grundy and Martin, 2001). However, they leave this conjecture untested. Our paper provides the explicit missing link between momentum returns and factor risk exposures.

The paper is organized as follows. Section 2 establishes evidence that price trends predict risk exposures, which lays the groundwork for our model. Section 3 describes our conditional factor model specification and the IPCA estimation approach. Section 4 presents our central finding that conditional risk exposures drive out the momentum effect, including a variety of robustness tests. Moreover, we find that conditional risk exposures also explain long-term reversal and part of short-term reversal. Section 5 explores the robustness of our results in several dimensions and Section 6 offers conclusions from our analysis.

2 Price Trends Predict Betas

In this section we document the robust stylized fact that a stock's recent price trends forecast its future realized betas on aggregate risk factors.

2.1 Data

Our primary sample is a large stock-level panel of returns and characteristics (see Table AIII in the appendix for the complete list of characteristics). This sample spans the CRSP and Compustat universe over the period 1966–2017. As in in Kelly et al. (2019), we transform all characteristics into cross-sectional ranks and normalize ranks to the [–0.5,0.5] interval. If at least three-fourths of characteristics are non-missing for a given stock-month observation, we include that observation and impute the remaining missing characteristics to take a value of zero (the cross-sectional median). This sample includes 22,812 unique stocks and 2,104,026 stock-month observations.

We face a a tradeoff in our data construction. On one hand, we prefer as many firm-month observations as possible. On the other hand, we prefer as many stock-level characteristics as possible to best inform our estimation of conditional betas. The larger the set of characteristics, the fewer the observations, because many characteristics are limited to the latter part of the sample or are missing for many firms. To demonstrate the robustness of our main findings, we also consider the dataset studied in Kelly et al. (2019), comprised of stock returns and 36 characteristics from Freyberger et al. (2017). That sample spans 1966–2014, restricts attention to stock-month observations for which all 36 characteristics are non-missing, and ultimately includes 12,813 unique stocks and 1,403,544 stock-month observations. We refer to this as the "KPS" sample, which has about 10,000 fewer unique stocks and 700,000 fewer stock-month observations (about 40% and 30% less, respectively) than our primary sample, but almost three times as many characteristics per observation.

2.2 Prediction Results

We construct stock-level monthly realized betas on the five Fama-French factors using daily data within month t. Likewise, we construct quaretly betas from daily data in months t to $t + 2$, and annual betas from daily data in months t to t+11.

We then explore the predictability of realized betas using recent stock momentum. In particular, we regress betas during months t to $t + k$ (k =1,3, or 12) on cumulative returns from month $t-12$ to $t-2$ in a stock-month panel (clustering standard errors by stock and by month).

The results, shown in Table I, demonstrate that stock momentum is a powerful predictor of future realized betas. The one-month predictive coefficient for market beta is 0.19 (t=9.7).

Table I
Predicting Realized Betas With Momentum

Notes – Standard errors clustered by month and firm. The Adjusted R^2 calculation is described in the appendix.

	Factor				
	MKTRF	SMB	HML	RMW	CMA
A: One-month					
Slope	0.19	−0.01	−0.08	0.18	−0.01
	(9.72)	(−0.54)	(−2.31)	(4.83)	(−0.17)
R^2 (%)	0.04	0.00	0.01	0.01	0.00
Adjusted R^2 (%)	7.44	0.01	1.86	1.86	0.02
B: Three-month					
Slope	0.18	−0.04	−0.04	0.24	−0.04
	(9.14)	(−1.69)	(−1.05)	(7.44)	(−0.95)
R^2 (%)	0.16	0.00	0.01	0.04	0.00
Adjusted R^2 (%)	9.83	0.05	0.61	2.46	0.06
C: Twelve-month					
Slope	0.14	−0.09	−0.10	0.24	−0.12
	(9.47)	(−4.80)	(−4.00)	(9.38)	(−3.90)
R^2 (%)	0.19	0.03	0.05	0.09	0.02
Adjusted R^2 (%)	3.97	0.63	1.04	1.88	0.42

This coefficient means that as a stock transitions from the 10^{th} to the 90^{th} momentum percentile, its market beta increases by 0.15 (0.19×0.8). The economic magnitude of this effect is large—it is median industry beta to the 75^{th} percentile of industry betas.[4] Momentum is also a strongly significant positive predictor of betas on the RMW profitability factor at every horizons that we analyze, with similar economic magnitudes to that for the market beta. It also significantly predicts 12-month betas on SMB, HML, and CMA.

It is difficult to interpret the R^2 in these regressions because the dependent variable suffers from severe measurement error. This mechanically depresses the R^2 compared to the infeasible regression of the true beta on lagged momentum. In Appendix A3 we devise a Monte Carlo approach to adjusting the monthly predictive R^2 for estimation error in realized betas. Our simulations suggest that roughly 95% of the variation in one-month realized betas estimated from daily is due to measurement error. After applying our R^2 correction for measurement error, we find that momentum explains approximately 7.4% of the panel

[4]Based on market beta estimates for 94 industries; see http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/totalbeta.html.

Table II
Predicting Realized Betas With Characteristics

Notes – Predictive regressions of one-month-ahead realized betas on stock-level characteristics. Standard errors clustered by month and firm. The Adjusted R^2 calculation is described in the appendix.

	Factor				
	MKTRF	SMB	HML	RMW	CMA
LOGME	0.71	0.05	−0.37	−0.57	0.02
	(22.13)	(1.15)	(−6.10)	(−8.42)	(0.32)
AT	0.04	−0.28	0.43	0.46	0.04
	(1.43)	(−7.24)	(7.45)	(7.10)	(0.53)
BETA	0.77	0.54	0.05	−0.17	−0.13
	(45.13)	(18.66)	(1.58)	(−4.35)	(−3.64)
BM	−0.01	−0.03	0.56	0.23	−0.14
	(−0.33)	(−1.49)	(16.70)	(5.78)	(−3.76)
EP	0.11	0.26	0.15	−0.05	0.03
	(6.67)	(11.60)	(5.20)	(−1.29)	(0.75)
GRPROF	0.01	0.01	−0.08	0.35	0.06
	(0.54)	(0.81)	(−3.72)	(12.23)	(2.29)
IVOL	0.28	0.38	−0.24	−0.40	0.06
	(13.62)	(11.28)	(−5.71)	(−8.32)	(1.08)
ROE	−0.08	−0.17	0.20	0.61	−0.19
	(−4.73)	(−7.10)	(6.24)	(15.89)	(−4.77)
ASSETG	0.01	0.07	0.03	−0.07	−0.54
	(1.68)	(5.80)	(2.02)	(−3.28)	(−25.89)
MOM12	0.18	0.12	−0.06	0.05	0.05
	(9.97)	(5.23)	(−1.77)	(1.33)	(1.21)
LTR	0.11	0.06	−0.04	−0.04	−0.15
	(8.20)	(3.52)	(−1.75)	(−1.54)	(−5.10)
STR	0.07	0.01	−0.00	0.02	0.00
	(4.54)	(0.53)	(−0.13)	(0.80)	(0.11)
W52H	−0.18	−0.07	−0.08	0.12	0.03
	(−8.71)	(−2.49)	(−2.06)	(2.85)	(0.62)
Constant	0.81	0.64	0.08	−0.14	0.04
	(163.24)	(96.77)	(9.67)	(−14.44)	(3.96)
R^2 (%)	1.90	0.46	0.25	0.24	0.10
Adjusted R^2(%)	54.95	13.30	7.23	6.94	2.89

variation in monthly realized market betas, and 1.9% in monthly realized RMW betas—the magnitudes rise to 9.8% and 2.5%, respectively, for three-month realized betas, and decrease at longer horizons.

Momentum is not unique in its ability to forecast future realized betas. Many commonly studied characteristics significantly forecast one-month-ahead betas in univariate regressions (see Table AII in Appendix A4).[5] In Table II, we report predictive coefficients in multiple re-

[5] We describe details of our data set in Section 4.1.

gressions of realized betas on all characteristics at once. The strongest market beta predictors are past realized betas and firm size. Even after accounting for these, price trend characteristics including momentum, long-term reversal, and trailing 52-week high remain highly significant for forecasting market beta, with coefficients of 0.18 (t=10.0), 0.11 (t=8.2), and −0.18 (t=−8.7), respectively. Additionally, price trends significantly predict future betas on SMB, HML, and CMA in multiple regression. Short-term reversal is a weaker forecaster of market beta than other price trends, with a coefficient of 0.07 (t=4.5), fails to significantly predict betas on other factors in multiple regression. The amount of monthly realized beta predictability is substantial, notably with adjusted R^2s of 55% on the market beta, 13% on SMB, 7% on both HML and RMW, and 3% on CMA.

The conclusion from these beta forecasting results is that so-called "anomaly" characteristics in general, and price trends in particular, are potent indicators of future realized risk. In other words, factor betas are significantly time-varying and stock characteristics are useful for tracking that beta variation. This hints at a route to reconciling the average return patterns associated with stock characteristics within a dynamic *conditional* model of the risk-return tradeoff, which we pursue next.

3 Model

We depart from prior literature by postulating a conditional factor pricing model for individual stocks, and investigating the viability of this model for explaining observed price trend effects for expected returns. Conditionality enters into our model via the specification of factor loadings, which we specify to be a function of observable firm characteristics. As a firm's characteristics evolve, so do its conditional risk exposures, and thus its model-implied expected returns. From the model's point of view, any persistence in stock performance—such as that captured with a momentum strategy—must originate from persistence in risk exposures.[6]

The primary empirical challenge to our analysis is estimating a conditional asset pricing model in order to evaluate its ability to capture the momentum effect. We use firm characteristics as instruments to help identify firms' otherwise hard-to-measure dynamic factor exposures. Within this "instrumented loading" specification, we explore two model variants. The first uses instrumented principal components analysis (IPCA), which treats factors as latent and estimates the factors that best associate with characteristics-based risk exposures.

[6]Further below we discuss the role for time-varying risk premia.

The second and more restrictive model relies on observable pre-specified factors such as the Fama-French factors.

3.1 Instrumented Principal Component Analysis

Kelly et al. (2019) provide a detailed analysis of the IPCA model, which we summarize here. They model the $N \times T$ panel of excess returns as

$$r_{i,t+1} = \underbrace{(z'_{i,t}\Gamma)}_{\beta_{i,t}} f_{t+1} + \tilde{\epsilon}_{i,t+1}. \tag{3}$$

This is a conditional pricing model. Assets are exposed to a set of K unobservable factors, which are denoted f_{t+1}. Section 2 demonstrates the strong association between stock-level characteristics and future betas. IPCA directly embeds this feature in within the specification of $\beta_{i,t}$. In particular, assets' dynamic conditional factor loadings may depend on observable asset characteristics contained in the $L \times 1$ instrument vector $z_{i,t}$ (which includes a constant). The $L \times K$ matrix Γ defines the mapping between a potentially large number of characteristics and a small number of risk factor exposures. In particular, Γ is the set of linear combinations of candidate characteristics that best predicts betas.[7] Latent factors and loadings in model (3) are estimated by minimizing the sum of squared model errors. Kelly et al. (2019) provide an efficient computational algorithm for estimation.

The IPCA mapping between characteristics and loadings provides a formal statistical bridge between characteristics and factor betas. The "restricted" form of the model shown in equation (2) also imposes that characteristics influence expected returns *only* because they determine betas. This form of the model fixes the factor model intercept (alpha) to zero, imposing the economic restriction that risk premia solely reflect compensation for systematic risk exposure. With this restriction, the model can only accommodate a price trend effect if it represents factor risk. Whether this is true is an empirical question. If momentum or other price trend effects are better described as anomaly alphas, the restricted model will be outperformed by an alternative model. We investigate such empirical tests below.

[7] Our model accommodates a potentially large set of conditioning instruments by performing a dimension reduction in the characteristic space. If there are many characteristics that provide noisy but informative signals about a stock's risk exposures, then aggregating characteristics into linear combinations isolates the signal and averages out the noise.

3.2 Instrumented Fama-French Model

The best overall conditional model studied by Kelly et al. (2019) uses latent factors. However, they also find that standard models with observable factors, such as the Fama-French five-factor model (Fama and French 2016), have dramatically improved performance when their loadings are instrumented with observable characteristics. Including characteristics in the specification of Fama-French loadings puts an observable factor model on the same informational footing as IPCA.

Thus, our second variation on model (3) replaces f_{t+1} with the five Fama-French factors, rather than treating f_{t+1} as latent. In this case, it is convenient to rewrite model (3) as

$$r_{i,t+1} = \text{vec}(\Gamma)'(f_{t+1} \otimes z_{i,t}) + \tilde{\epsilon}_{i,t+1}. \tag{4}$$

The term $f_{t+1} \otimes z_{i,t}$ is the $KL \times 1$ vector of each factor interacted with each characteristic. Because the factors are observable, an OLS regression of returns onto the factor/characteristic interactions recovers Γ, and in turn recovers the conditional loadings, $\beta_{i,t}$. From this it is clear that Γ is determined by a panel OLS regression of individual returns on aggregate factors interacted with lagged characteristics.[8]

Our main IPCA analyses use a five-factor IPCA specification. This choice is motivated by the analysis in Kelly et al. (2019) and makes our model comparable with the Fama-French five-factor specification that we study.

3.3 Models and Price Trends

To investigate the ability of an asset pricing model to capture the momentum effect, we analyze three competing return predictors, each of which proxies for the true conditional expected asset return, $E_t[r_{i,t+1}]$.

The first predictor is the traditional momentum signal. This uses each assets' recent past return performance—defined as a moving average of prior returns—as a signal for conditional expected future returns. This predictor is agnostic of the model. Our primary momentum construction estimates $E_t[r_{i,t+1}]$ with $\bar{r}_{i,t} = \sum_{j=1}^{12} r_{i,t-j}$, which is the standard 2–12 momentum filter originally proposed by Asness (1994) and popularized by Carhart (1997).

Second is the model-based predictor, defined as the conditional expectation of the factor

[8] Write $\tilde{z}_{i,t+1} \equiv f_{t+1} \otimes z_{i,t}$ and then $\text{vec}(\hat{\Gamma})$ is given by $(\sum_{i,t} \tilde{z}_{i,t+1}\tilde{z}'_{i,t+1})^{-1} \sum_{i,t} \tilde{z}_{i,t+1} r_{i,t+1}$.

component of returns, $\beta'_{i,t}\lambda_t$, where $\lambda_t = E_t[f_{t+1}]$. To focus squarely on the role of time-varying risk exposures, most of our analysis treats the expected factor return as constant: $E_t(F_{t+1}) = \lambda$. We return the topic of time-varying expected factor returns in the robustness analysis of Section 5.4.

The third predictor is recent *unexplained* stock performance—defined as a moving average of model residuals $\epsilon_{i,t}$ defined in (2). This is a direct analogue of momentum, but applied to residual stock returns after controlling for the conditional factor model. Our residual momentum construction mirrors that of raw momentum and proxies for $E_t[r_{i,t+1}]$ with $\bar{\epsilon}_{i,t} = \sum_{j=1}^{12} \epsilon_{i,t-j}$. Notice that we build this signal from errors relative to the *conditional expectation* provided by (2), not from the idiosyncratic returns in (3) or (4). The difference is that residuals from the conditional expectation also contain shock realization of the factors $f_{t+1} - \lambda_t$, while the true idiosyncratic return would have the full contemporaneous realization of f_{t+1} stripped out. Thus residual momentum not only captures momentum from the idiosyncratic returns but also any momentum in the factors themselves (e.g., Gupta and Kelly, 2018). Thus, our residual momentum calculation is an especially conservative test of factor model's ability to capture stock momentum.

By testing the comparative return prediction performance of these three variables we can ascertain the success or failure of the asset pricing model in describing the momentum anomaly. The predictive content of $\bar{r}_{i,t}$ establishes the baseline momentum effect in our sample. If an asset pricing model successfully explains the momentum effect, then all of the predictive content from raw momentum should be attributable to $\beta'_{i,t}\lambda_t$, and $\bar{\epsilon}_{i,t}$ will have zero incremental predictive power. If instead the model does a poor job of capturing momentum, then the signal content of stock momentum will be inherited by residuals and as a result residual momentum will enter as a significant predictor.

We conduct prediction tests in a variety of ways. First, we run panel predictive regressions of $r_{i,t+1}$ on $\bar{r}_{i,t}$, $\beta'_{i,t}\lambda_t$, or $\bar{\epsilon}_{i,t}$. We also consider predictive regressions that instead use the cross-sectional ranks of each of these predictors, which is a closer counterpart to the often studied cross-sectional sorts in the literature. Likewise, to easily compare our findings with the prior literature, we analyze trading strategies that sort stocks into equal-weighted portfolios based on each signal to track and compare the signal performance.

4 Can A Conditional Model Explain Price Trend Facts?

We evaluate IPCA's ability to explain momentum in a host of specifications and datasets, both in-sample and out-of-sample. We investigate how our findings differ when using a range of alternative formation windows other than 2–12, and find that our model-based conditional expectations explain a large portion of momentum and long-term reversal phenomena. We also show that short-term reversal is a distinct phenomenon not captured by IPCA.

4.1 Data

Our primary sample is a large stock-level panel of returns and characteristics. This sample spans the CRSP and Compustat universe over the period 1966–2017. As in in Kelly et al. (2019), we transform all characteristics into cross-sectional ranks and normalize ranks to the [–0.5,0.5] interval. If at least three-fourths of characteristics are non-missing for a given stock-month observation, we include that observation and impute the remaining missing characteristics to take a value of zero (the cross-sectional median). This sample includes 22,812 unique stocks and 2,104,026 stock-month observations.

We face a a tradeoff in our data construction. On one hand, we prefer as many firm-month observations as possible. On the other hand, we prefer as many stock-level characteristics as possible to best inform our estimation of conditional betas. The larger the set of characteristics, the fewer the observations, because many characteristics are limited to the latter part of the sample or are missing for many firms. To demonstrate the robustness of our main findings, we also consider the dataset studied in Kelly et al. (2019), comprised of stock returns and 36 characteristics from Freyberger et al. (2017). That sample spans 1966–2014, restricts attention to stock-month observations for which all 36 characteristics are non-missing, and ultimately includes 12,813 unique stocks and 1,403,544 stock-month observations (see Table AIII in the appendix for the complete list of characteristics). We refer to this as the "KPS" sample, which has about 10,000 fewer unique stocks and 700,000 fewer stock-month observations (about 40% and 30% less, respectively) than our primary sample, but almost three times as many characteristics per observation.

4.2 IPCA Model

Table III reports our main empirical result. It analyzes the performance of the IPCA model in explaining the momentum effect using our primary sample.

Table III
Momentum and the IPCA Model

Notes – This table uses the primary data sample. Panels A and C report coefficient estimates, t-statistics, and R^2 (in percentage) of univariate and bivariate panel regressions, respectively, of the next month's excess stock returns on the current month's signal (left three columns) or signal rank (right three columns). t-statistics use standard errors clustered by month. Panel B reports annualized average returns (left three columns) and Sharpe ratios (right three columns) of equal-weighted quintile portfolios (Q1, Q2, etc.) sorted on each of the three signals. The row labeled Q5–Q1 denotes the spread portfolio that is long Q5 and short Q1, and t-statistics are provided for these averages and Sharpe ratios. Average returns are reported in annualized percentage. Slope coefficients in regressions using ranks are multiplied by 100 for readability.

A. Univariate Regressions

	Raw signal			Rank signal		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Constant	0.01	0.00	0.01	0.01	0.01	0.01
(t-stat)	(3.87)	(0.74)	(4.05)	(4.06)	(4.04)	(4.06)
Coeff	0.00	0.86	−0.00	0.87	1.92	0.72
(t-stat)	(0.12)	(11.48)	(−0.04)	(3.39)	(11.10)	(2.83)
R^2 (%)	0.00	0.13	0.00	0.02	0.12	0.02

B. Portfolio Sorts

	Average return			Sharpe ratio		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Q1	5.97	0.01	6.81	0.23	0.00	0.27
Q2	8.11	6.82	8.62	0.44	0.38	0.46
Q3	9.90	9.88	9.55	0.59	0.54	0.57
Q4	11.87	13.77	11.33	0.69	0.70	0.66
Q5	14.99	20.36	14.51	0.68	0.94	0.66
Q5–Q1	9.02	20.35	7.69	0.53	1.71	0.45
(t-stat)	(3.75)	(12.21)	(3.22)	(3.73)	(11.54)	(3.04)

C. Bivariate Regressions

	Raw signal			Rank signal		
	1	2	3	4	5	6
Constant	0.00	0.00	0.00	0.01	0.01	0.01
(t-stat)	(0.68)	(0.64)	(1.93)	(4.04)	(4.04)	(4.06)
$\bar{r}$	−0.00		0.04	0.29		5.25
(t-stat)	(−0.73)		(4.95)	(1.01)		(7.37)
$\beta'\lambda$	0.92	0.90		1.82	1.85	
(t-stat)	(8.75)	(9.85)		(8.68)	(9.61)	
$\bar{\epsilon}$		−0.00	−0.04		0.31	−4.45
(t-stat)		(−0.66)	(−5.07)		(1.15)	(−6.50)
R^2 (%)	0.14	0.14	0.03	0.12	0.12	0.04

Panel A of Table III reports univariate panel predictive regressions of the form

$$r_{i,t+1} = c_0 + c_1 s_{i,t} + e_{i,t+1},$$

where $s_{i,t}$ represents either 2–12 return momentum ($\bar{r}$), the model-based expected return ($\beta'\lambda$), or momentum in model residuals ($\bar{\epsilon}$)—we drop the inessential t subscripts from here on. In all panel regressions, we cluster standard errors by month to account for cross-sectional correlation in returns. An ideal proxy for the true conditional expected return would have an intercept of zero, a slope of one, and a comparatively high R^2 (Mincer and Zarnowitz, 1969). While this is possible for the raw predictors, it will not be the case for ranked predictors due to their scaling, in fact showing a limitation of ranked predictors vis-à-vis basic prediction tests.

The "$\bar{r}$" columns of Panel A establish the baseline behavior of stock momentum in the primary sample. It is interesting to note that the *raw* return momentum signal has no predictive power for future returns. Both it's predictive slope coefficient and panel R^2 are almost exactly zero. Only after it is converted into a cross-sectional rank does return momentum predict future returns, in which case the monthly panel R^2 on past momentum rank is 0.02%, with a significantly positive slope coefficient (t=3.4). This low predictive R^2 nonetheless translates into potent trading strategy performance, as indicated in the portfolio sorts of Panel B. Sorting on $\bar{r}$ produces a Q5–Q1 spread with an annualized average return of 9.0% (t = 3.8) and Sharpe ratio of 0.53 (t = 3.7). Sharpe ratio t-statistics are based on the asymptotic standard error formula of Lo (2002).

In comparison with return momentum, the model-based conditional expected return ("$\beta'\lambda$" columns of Panel A) has much stronger predictive power. In its raw form, the predictive slope is 0.86 (t=11.5). While this slope is significantly different from zero at the 5% level, it is insignificantly different from 1.0, and the intercept is a indistinguishable from zero. Thus we *cannot* reject the hypothesis that the model gives unbiased estimates of conditional expected stock returns. The corresponding monthly panel R^2 is 0.13%, or nearly six times higher than the ranked version of return momentum. And, unlike momentum, converting $\beta'\lambda$ to a cross-sectional rank *slightly weakens* its predictive signal. Nonetheless, forming a Q5–Q1 portfolio spread based on the model's (in-sample) conditional expectation estimate produces an annualized average return of 20.4% (t = 12.2) and Sharpe ratio of 1.71 (t = 11.5).

Finally, we consider momentum in IPCA prediction residuals. In univariate analysis, the predictive power of residual momentum is somewhat weaker than that of total return momentum. The univariate slope coefficient on the ranked signal falls to 0.72 (t = 2.8) and the

Q5–Q1 Sharpe ratio falls to 0.45 ($t = 3.0$). Turning to Panel B, we find the exact opposite of what Grundy and Martin (2001) obtained: our factor-model-based strategy is far more profitable than raw momentum, while the residual strategy is modestly less profitable.

Univariate tests indicate that the model produces much more potent return predictions than simple momentum signals. But comparisons of univariate results do not offer a test of competing models. A more direct test of the conditional model's ability to explain momentum is to conduct joint predictive regressions controlling for momentum and the model-based expected return simultaneously.

These bivariate regressions are shown in Panel C. We find that the predictive content of momentum is mostly subsumed by the model-based expected return. Controlling for the conditional model, the momentum signal loses significance and can even switch sign. This is true whether we compare against raw or cross-sectionally ranked return momentum (columns 1 and 4). Columns 2 and 5 provide a complete decomposition of the total return into the model-based conditional expectation and a residual. In contrast to Grundy and Martin (2001), we find weak evidence of residual momentum. In the ranked case, it's coefficient estimate drops from 0.72 standalone to 0.31 when controlling for $\beta'\lambda$ and is insignificantly different from zero (t=1.2). Instead, our results indicate that conditional expected returns, derived from dynamic factor exposures, are primarily responsible for the predictability embodied by momentum. In Section 5 we present novel evidence from realized covariances lending support for a risk-based interpretation of these factor exposures.

4.3 IPCA Out-of-sample

The timing of our model ensures that conditioning characteristics entering into $\beta_{i,t}$ are fully known to market participants at or before time t. The estimates of the static parameter matrix Γ, however, use information from the entire sample. So, while there is no look-ahead bias in Table III, the model-based results are in-sample predictions.

In Table IV, we conduct the same comparative predictive analysis using entirely out-of-sample model-based prediction. In particular, when forecasting realized return $r_{i,t+1}$, we estimate Γ using data only through date t. Our initial estimation window uses data from January 1966 to June 1971 to forecast the July 1971 return. In each subsequent month, we recursively re-estimate the model and construct forecasts using an expanding, backward-looking sample. Small differences in performance of total return momentum between Tables III and IV are due to the difference between full sample (1966-2014) and out-of-sample

Table IV
Momentum and the IPCA Model (Out-of-sample)

Notes – This table uses the primary data sample. Model-based conditional expected returns are from recursive out-of-sample estimation. See Table III for further table description.

A. Univariate Regressions

	Raw signal			Rank signal		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Constant	0.01	0.00	0.01	0.01	0.01	0.01
(t-stat)	(3.80)	(2.09)	(3.97)	(3.99)	(3.98)	(3.99)
Coeff	0.00	0.55	0.00	0.87	1.76	0.73
(t-stat)	(0.12)	(7.48)	(0.06)	(3.27)	(9.69)	(2.79)
R^2 (%)	0.00	0.09	0.00	0.02	0.10	0.02

B. Portfolio Sorts

	Average return			Sharpe ratio		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Q1	6.34	0.84	7.04	0.25	0.04	0.28
Q2	8.54	7.68	9.07	0.46	0.43	0.49
Q3	10.26	10.64	9.95	0.62	0.59	0.59
Q4	12.29	13.78	11.71	0.72	0.72	0.68
Q5	15.06	19.55	14.73	0.68	0.90	0.67
Q5–Q1	8.71	18.71	7.69	0.50	1.54	0.45
(t-stat)	(3.41)	(10.46)	(3.02)	(3.40)	(9.98)	(2.88)

C. Bivariate Regressions

	Raw signal			Rank signal		
	1	2	3	4	5	6
Constant	0.00	0.00	0.01	0.01	0.01	0.01
(t-stat)	(2.09)	(2.08)	(2.96)	(3.99)	(3.99)	(4.00)
$\bar{r}$	−0.00		0.01	0.38		4.12
(t-stat)	(−0.60)		(0.91)	(1.30)		(5.42)
$\beta'\lambda$	0.58	0.56		1.64	1.67	
(t-stat)	(6.87)	(7.21)		(7.69)	(8.39)	
$\bar{\epsilon}$		−0.00	−0.01		0.38	−3.31
(t-stat)		(−0.46)	(−0.92)		(1.35)	(−4.49)
R^2 (%)	0.10	0.10	0.00	0.10	0.10	0.04

(1971-2014) evaluation periods.

The out-of-sample prediction power of IPCA's conditional expectations is very similar to our in-sample findings. The univariate out-of-sample R^2 is 0.09-0.10% for the raw and ranked versions of $\beta'\lambda$. The univariate intercept is significant but economically small, indicating a small bias in the model's out-of-sample predictions. However, the slope coefficient on (unranked) $\beta'\lambda$ is now 0.55 and is significantly different from 1.0. This indicates an attenuation bias in out-of-sample $\beta'\lambda$, presumably due to less data being is used to construct each forecast.

The central result of Table IV is that the model continues to fully subsume the momentum effect even when the model is estimated on an entirely out-of-sample basis. As in Panel C of Table III, the predictive coefficients on $\bar{r}$ and $\bar{\epsilon}$ are small and insignificant in either the raw or ranked signal cases (t=−0.6 and t=1.3, respectively).

Meanwhile, in Panel B we find that the impressive performance of model-based predictions in a simulated trading strategy are *not* an artifact of in-sample overfit. While there is minor attenuation of the out-of-sample Q5–Q1 annualized average return of 18.7% and Sharpe ratio of 1.54 (compared to an in-sample 20.4% and 1.71, respectively), it remains a more than two-fold improvement on return momentum.

The close similarity between Tables III and IV indicate that our main findings in Table III are not driven by in-sample biases.

4.4 KPS Stock Sample

Our primary sample is deliberately constructed to include the largest possible number of stock-month observations. This raises the question of whether our results thus far are driven by the preponderance of small firms, for instance if their stocks are less liquid and more prone to return trends.

The KPS sample, in contrast, is designed to incorporate a larger collection of firm characteristics. Filtering out firms with incomplete characteristic data skews the sample toward larger and longer established firms. These stocks may exhibit less momentum to begin with. On the other hand, the KPS sample includes more conditioning information which may make our estimates of time-varying risk exposure more precise.[9]

Table V reports the comparison of IPCA versus momentum using the KPS data. The baseline momentum effect in the KPS sample is only slightly weaker than in the broader sample—in

[9] Appendix Table **??** shows results using the KPS data but restricted to only using the thirteen characteristics in the Primary data set: the results are quite similar to what Table III reports.

Table V
Momentum and the IPCA Model (KPS Sample)

Notes – This table uses the KPS data sample. See Table III for further table description.

A. Univariate Regressions

	Raw signal			Rank signal		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Constant	0.01	0.00	0.01	0.01	0.01	0.01
(t-stat)	(3.59)	(0.13)	(3.70)	(3.70)	(3.70)	(3.70)
Coeff	−0.00	0.99	−0.00	0.72	3.24	0.59
(t-stat)	(−0.29)	(14.02)	(−0.42)	(2.52)	(13.91)	(2.07)
R^2 (%)	0.00	0.37	0.00	0.02	0.32	0.01

B. Portfolio Sorts

	Average return			Sharpe ratio		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Q1	7.96	−4.59	8.70	0.30	−0.22	0.32
Q2	8.59	5.59	9.29	0.43	0.29	0.46
Q3	10.26	9.75	10.05	0.55	0.49	0.54
Q4	12.65	15.93	11.82	0.67	0.75	0.63
Q5	16.25	29.01	15.82	0.69	1.14	0.68
Q5–Q1	8.29	33.60	7.12	0.48	2.39	0.41
(t-stat)	(3.30)	(16.48)	(2.84)	(3.29)	(14.82)	(2.56)

C. Bivariate Regressions

	Raw signal			Rank signal		
	1	2	3	4	5	6
Constant	0.00	−0.00	0.01	0.01	0.01	0.01
(t-stat)	(0.11)	(−0.08)	(2.07)	(3.70)	(3.70)	(3.70)
$\bar{r}$	−0.01		0.04	−0.14		4.63
(t-stat)	(−1.45)		(3.24)	(−0.42)		(6.08)
$\beta'\lambda$	1.06	1.04		3.27	3.25	
(t-stat)	(11.83)	(12.41)		(11.88)	(12.39)	
$\bar{\epsilon}$		−0.01	−0.04		−0.08	−3.97
(t-stat)		(−1.38)	(−3.71)		(−0.26)	(−5.32)
R^2 (%)	0.40	0.40	0.02	0.32	0.32	0.03

Panel A, the univariate slope coefficient on ranked momentum falls to 0.72 (t=2.5), from 0.87 (t=3.4) in the primary sample. In Panel B, the Q5–Q1 Sharpe ratio falls to 0.48 (t=3.3) from 0.53 (t=3.7). This is consistent with the fact that momentum tends to be stronger among smaller, younger, and more volatile firms.

However, the conditional model's predictive power notably *increases*, reflecting the richer conditioning information available to estimate conditional risk exposures. The qualitative conclusions from Table V are identical to those from Tables III and IV. In Panel A's univariate $\beta'\lambda$ regressions, the intercept is insignificantly different from zero. The slope is 0.99 (t=14.0) and is insignificantly different from 1.0, indicating that IPCA conditional expected returns are unbiased in the KPS sample as well. In Panel B, the model-based Q5–Q1 spread again generates an impressive annualized average return and Sharpe ratio of 33.6% (t=16.5) and 2.4 (t=14.8), respectively.

The key empirical result of Table III—that the momentum effect is in large part explained by conditional risk exposures—is also corroborated in the KPS sample. Momentum fares worse here, at least in terms of cross-sectionally ranked signals, as Panel C's bivariate coefficients on $\bar{r}$ and $\bar{\epsilon}$ become negative after controlling for $\beta'\lambda$, though they remain insignificantly different from zero ($t = -0.4$ and $t = -0.3$, respectively).[10]

Finally, note that across Tables III-V the $\beta'\lambda$ strategy has a much more profitable Q5 portfolio than $\bar{r}$. This says that even a *long-only strategy* employing the model-based conditional expectations out-performs the high momentum portfolio. This consistent result alleviates any possible concerns that our conclusions rely on being able to short Q1.[11]

4.5 Other Formation Windows

Thus far, we have focused on IPCA's ability to capture the momentum effect via a conditional factor model. Of the other various price trend phenomena documented in the literature (including short-term and long-term reversal), 2–12 momentum has received the most attention due to its strength, robustness, and its survival of trading costs (Asness et al. 2014).

In Table VI, we revisit other commonly studied price trend predictors vis-a-vis the IPCA conditional factor model. In addition to the traditional momentum strategy based on average returns over 2–12 months prior to portfolio formation, we consider signals based on returns over the 13–24 and 25–36 months prior to portfolio formation (long-term reversal of De Bondt and Thaler, 1985) and returns over the most recent one month (short-term reversal

[10]Out-of-sample results using the KPS data sample are in Table AIV in the appendix, and echo the qualitative points made here.

[11]Appendix Table **??** double-sorts 5x5 portfolios based on $\beta'\lambda$ and $\bar{r}$. It shows that the model-based $\beta'\lambda$ spread strategy robustly exhibits superior performance (an annualized Sharpe ratio higher than 0.6) to momentum within every momentum quintile, for both data sets.

Table VI
Other Formation Windows

Notes – Regression results using alternative windows to form $\bar{r}$, capturing long-term and short-term reversals, within the primary data sample.

		Rank signal regressions				
		Univariate		Bivariate		
Formation		$\bar{r}$	R^2 (%)	$\bar{r}$	$\beta'\lambda$	R^2 (%)
A. In-sample						
2	12	0.87	0.02	0.29	1.82	0.12
		(3.39)		(1.01)	(8.68)	
13	24	−0.46	0.01	−0.16	1.80	0.11
		(−2.62)		(−0.91)	(10.84)	
25	36	−0.26	0.00	0.05	1.78	0.11
		(−1.77)		(0.37)	(11.02)	
1	1	−1.70	0.09	−0.92	1.52	0.14
		(−7.07)		(−3.21)	(7.13)	
B. Out-of-sample						
2	12	0.87	0.02	0.38	1.64	0.10
		(3.27)		(1.30)	(7.69)	
13	24	−0.43	0.01	−0.13	1.67	0.10
		(−2.40)		(−0.72)	(9.56)	
25	36	−0.28	0.00	0.04	1.63	0.09
		(−1.81)		(0.25)	(9.71)	
1	1	−1.69	0.09	−0.87	1.29	0.11
		(−6.82)		(−2.45)	(4.50)	

of Jegadeesh, 1990). All of these price trend signals are only predictive when they are cross-sectionally ranked, therefore we focus our comparisons on panel predictive regressions using ranked signals. For each signal, we report the univariate panel regression, as well as the bivariate regression that controls for the model-based conditional expected return. There is no re-estimation of the IPCA model across the formation windows—we are holding fixed the estimated $\beta'\lambda$ and investigating whether this estimate of the conditional expected return drives out the predictability contained in various technical indicators.

Panel A reports results from the primary sample. Confirming well known results from the literature, long-term reversal (defined as performance either two or three years prior to portfolio formation) and short-term reversal enter with significant negative predictive coefficients. However, the bivariate regression shows that model-based risk compensation subsumes the predictive effects of *every* longer-term price trend signal. Momentum and both versions of long-term reversal become statistically insignificant. Furthermore, the bivariate R^2 is essentially equal to the univariate R^2 for $\beta'\lambda$ (seen in Table III), indicating that the predictive content in momentum and long-term reversal is mostly subsumed by the conditional factor model.

Meanwhile, the magnitude of short-term reversal's coefficient is cut in half from -1.7 ($t = -7.1$) to -0.9 ($t = -3.2$). High-frequency reversal is at least partially driven by illiquidity of small firms (Asness et al. 2014), so it is perhaps unsurprising that it survives in a model that abstracts from transaction costs. In Panel B, we re-administer these tests using our recursive out-of-sample estimates of model-based conditional expectations. The conclusions are identical to Panel A.

The results for long-term reversal are consistent with our basic model of conditional expectations as dynamic and persistent. It is counterintuitive that long-term reversal might emerge in a system with positive serial correlation. In the appendix we show that long-term reversal (that is, a negative and significant t-statistic on average returns over the somewhat distant past) can appear in a non-trivial proportion (10–20%) of simulations from this model. Hence, a more powerful test of long-term reversal comes from including the persistent conditional expectation along with the long-term average—this would uncover that the latter is statistically insignificant and indeed tells us nothing unique about future returns. This is exactly the bivariate regression we run, and we find evidence supporting the model's view of the world.

5 Robustness

In this section we consider the robustness of our results in several dimensions. First, we exclude momentum itself as an instrument, and analyze its comparative importance depending on the richness of the overall instrumental conditioning information. Second, we estimate the instrumented Fama-French model and see whether conditional risk exposures continue to explain a substantial portion of the momentum strategy's profitability, when using unconditional risk factors pre-specified outside of the model. Third, we provide a novel method of estimating a dynamic λ_t in our latent-factor IPCA model and evaluate what role time-varying risk premia, from which we have abstracted until now, play in explaining momentum returns. Fourth, we turn to daily data to support our risk-based interpretation with novel out-of-sample analysis, finding that our dynamic beta estimates are good predictors of future returns' covariance with aggregate factors. Fifth, we document that our model-based $\beta'\lambda$ strategy does not crash (cf. Daniel and Moskowitz 2016) and exhibits less higher-moment risk than does momentum. Finally, we consider the impact of value weighting our portfolios and consider the turnover inherent in various strategies.

5.1 Excluding Momentum From IPCA

In our IPCA specifications thus far, momentum itself is an instrument in the beta specification. The model tightly restricts the use of characteristics—they can only enter as a means of capturing conditional covariances among returns. The evidence of Section 2 unambiguously indicates that momentum is a powerful predictor of factor exposures, justifying its role as an instrument for beta. By deliberately fixing alphas to zero, the model shuts down any role of characteristics *other than* their role in determining factor betas. Nevertheless, one may wonder if momentum is the only useful characteristic to this end?

Table VII (which uses the primary sample) suggests that our previous findings are minimally altered by excluding momentum as a beta-conditioning characteristic. Panel A of Table VII reports that the model-based raw signal remains an unbiased prediction of future returns, with an insignificant intercept ($t = 0.6$) and slope coefficient of 0.92 that is statistically indistinguishable from 1.0. Panel B reports that the model-based trading strategy remains the best performer even without momentum itself entering the model. In Table VII the annualized Sharpe ratio is 1.2 ($t = 8.3$), which is less than the 1.7 we found in Table III but is still more than double the 0.5 achieved by the basic momentum strategy.

The biggest change in results from excluding momentum appears in the bivariate regressions of Panel C. Column 5 shows that momentum, defined either as $\bar{r}$ or $\bar{\epsilon}$, remains statistically significant after controlling for $\beta'\lambda$. While this is true in our primary sample, the answer is somewhat different in the KPS universe. Panel C, Column 5 in Table VIII shows that $\bar{r}$ and $\bar{\epsilon}$ are positive but statistically insignificant after controlling for $\beta'\lambda$.

This suggests that excluding momentum as an instrument is relatively more important in our primary data sample than in the KPS data sample. The reason for this is intuitive: our primary sample employs only 13 characteristics (in order to retain more stock-month observations) while the KPS sample employs 36 characteristics. Including a larger set of characteristics helps substitute for the loss of momentum as a beta predictor. In other words, much of the beta predictive power of momentum can be captured by other characteristics, as long as enough characteristics are considered as in the case of the KPS sample. Furthermore,

Table VII
IPCA Excluding the Momentum Characteristic

Notes – See Table III.

A. Univariate Regressions

	Raw signal			Rank signal		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Constant	0.01	0.00	0.01	0.01	0.01	0.01
(t-stat)	(3.87)	(0.55)	(4.05)	(4.06)	(4.04)	(4.06)
Coeff	0.00	0.92	−0.00	0.87	1.52	0.71
(t-stat)	(0.12)	(7.88)	(−0.06)	(3.39)	(7.94)	(2.75)
R^2 (%)	0.00	0.09	0.00	0.02	0.07	0.02

B. Portfolios

	Average return			Sharpe ratio		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Q1	5.97	2.78	6.87	0.23	0.14	0.27
Q2	8.11	7.18	8.62	0.44	0.41	0.46
Q3	9.90	9.29	9.64	0.59	0.50	0.57
Q4	11.87	12.84	11.30	0.69	0.65	0.65
Q5	14.99	18.73	14.41	0.68	0.84	0.65
Q5–Q1	9.02	15.94	7.55	0.53	1.20	0.44
(t-stat)	(3.75)	(8.56)	(3.12)	(3.73)	(8.32)	(3.03)

C. Bivariate Regressions

	Raw signal			Rank signal		
	1	2	3	4	5	6
Constant	0.00	0.00	0.00	0.01	0.01	0.01
(t-stat)	(0.48)	(0.48)	(1.20)	(4.04)	(4.04)	(4.06)
$\bar{r}$	0.00		0.06	0.96		7.32
(t-stat)	(0.25)		(5.90)	(3.75)		(8.55)
$\beta'\lambda$	0.92	0.93		1.57	1.68	
(t-stat)	(8.18)	(8.29)		(8.45)	(9.37)	
$\bar{\epsilon}$		0.00	−0.06		0.98	−6.52
(t-stat)		(0.30)	(−6.17)		(3.91)	(−7.65)
R^2 (%)	0.09	0.09	0.04	0.10	0.10	0.06

the KPS universe is comprised of larger and longer-lived firms, which have a smaller baseline momentum effect to begin with.

These results help summarize our main finding: Momentum is a simple filter that effectively

Table VIII
IPCA Excluding the Momentum Characteristic, KPS Sample

Notes – See Table III.

A. Univariate Regressions

	Raw signal			Rank signal		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Constant	0.01	0.00	0.01	0.01	0.01	0.01
(t-stat)	(3.59)	(0.04)	(3.70)	(3.70)	(3.70)	(3.70)
Coeff	−0.00	1.00	−0.00	0.72	3.12	0.58
(t-stat)	(−0.29)	(13.19)	(−0.43)	(2.52)	(13.06)	(2.03)
R^2 (%)	0.00	0.36	0.00	0.02	0.30	0.01

B. Portfolios

	Average return			Sharpe ratio		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Q1	7.96	−3.58	8.82	0.30	−0.18	0.33
Q2	8.59	5.52	9.02	0.43	0.29	0.45
Q3	10.26	9.90	10.27	0.55	0.50	0.55
Q4	12.65	14.96	11.84	0.67	0.70	0.63
Q5	16.25	28.91	15.76	0.69	1.13	0.67
Q5–Q1	8.29	32.49	6.94	0.48	2.29	0.40
(t-stat)	(3.30)	(15.72)	(2.76)	(3.29)	(14.26)	(2.50)

C. Bivariate Regressions

	Raw signal			Rank signal		
	1	2	3	4	5	6
Constant	0.00	0.00	0.01	0.01	0.01	0.01
(t-stat)	(0.11)	(0.01)	(1.79)	(3.70)	(3.70)	(3.70)
$\bar{r}$	−0.00		0.04	0.42		5.69
(t-stat)	(−0.79)		(3.39)	(1.42)		(6.91)
$\beta'\lambda$	1.02	1.01		3.08	3.10	
(t-stat)	(12.42)	(12.84)		(12.28)	(12.74)	
$\bar{\epsilon}$		−0.00	−0.04		0.45	−5.03
(t-stat)		(−0.72)	(−3.98)		(1.56)	(−6.14)
R^2 (%)	0.37	0.36	0.02	0.30	0.31	0.04

captures time-varying expected returns, whatever their origin. The momentum characteristic contributes relevant conditioning information for dynamic betas within the IPCA framework. When IPCA's information set is sufficiently rich, the momentum characteristic adds relatively little incremental information for beta (hence the KPS sample results). When that information set is poorer, the momentum characteristic provides a lot of incremental information for beta prediction (hence the primary sample results).

5.2 Instrumented Fama-French Model

Our analysis has centered on conditional expected returns from a latent factor model. The basic insight of using stock characteristics to instrument for conditional factor loadings can likewise be applied to pre-specified observable factors. As described in equation (4), the conditional loadings are estimated as a regression of returns onto factors interacted with firm characteristics. In this simple regression setting it is more transparent that our model only loads on a characteristic if it is helpful for describing betas. As a regression-based estimator, the model is agnostic to the return prediction power of characteristics per se. The least squares objective function only wants to include characteristics in the model if they improve measurement of factor covariances. While this is equally true in the latent factor IPCA optimization, it is perhaps more clearly illustrated in this observable factor setting.

In Table IX, we investigate whether the five-factor Fama-French (2015) model, modified to incorporate instrumented conditional loadings, can capture the momentum effect in our primary data set. Panel A provides a first comparison of the conditional version of Fama-French to latent factor IPCA. The raw signal regression shows that, like IPCA, expected returns from the conditional Fama-French model are much stronger univariate predictors than momentum. However, the univariate Fama-French R^2 is smaller that from IPCA: 0.05% per month, or roughly one-third of that shown in Tables III. Furthermore, the conditional Fama-French model produces biased estimates of conditional expected returns—in the raw $\beta'\lambda$ regression, the intercepts are significantly negative and the slope coefficients are significantly greater than one.

Similarly, Panel B shows that the model-based spread portfolios are more profitable than the momentum strategy. The conditional expected returns ($\beta'\lambda$) generate a Sharpe ratio of 1.0 ($t = 7.3$), doubling that of the momentum signal. This performance, however, is notably smaller than what is obtained in Table III using the IPCA model.

The conditional Fama-French model captures part of the momentum effect, but momentum remains an important predictor above and beyond model-based return forecasts. For instance, in the bivariate rank signal regressions in Panel C of Table IX, $\beta'\lambda$ has a t-statistic of 5.7 while residual momentum has a t-statistic of 2.3, and the magnitudes of momentum

Table IX
Momentum and the Instrumented Fama-French Model

Notes – See Table III. Model uses characteristic-based conditional loadings on the five Fama-French factors.

A. Univariate Regressions

	Raw signal			Rank signal		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Constant	0.01	−0.01	0.01	0.01	0.01	0.01
(t-stat)	(3.87)	(−2.56)	(4.03)	(4.06)	(4.00)	(4.06)
Coeff	0.00	2.02	0.00	0.87	1.27	0.85
(t-stat)	(0.12)	(6.46)	(0.09)	(3.39)	(7.03)	(3.29)
R^2 (%)	0.00	0.05	0.00	0.02	0.05	0.02

B. Portfolio Sorts

	Average return			Sharpe ratio		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Q1	5.97	3.22	6.05	0.23	0.18	0.24
Q2	8.11	8.25	8.31	0.44	0.49	0.45
Q3	9.90	10.16	9.63	0.59	0.57	0.58
Q4	11.87	13.07	11.85	0.69	0.65	0.69
Q5	14.99	16.14	14.98	0.68	0.65	0.68
Q5–Q1	9.02	12.92	8.93	0.53	1.04	0.52
(t-stat)	(3.75)	(7.41)	(3.70)	(3.73)	(7.25)	(3.62)

C. Bivariate Regressions

	Raw signal			Rank signal		
	1	2	3	4	5	6
Constant	−0.01	−0.01	0.00	0.01	0.01	0.01
(t-stat)	(−2.44)	(−2.40)	(0.94)	(4.02)	(4.02)	(4.06)
$\bar{r}$	−0.00		0.08	0.63		8.89
(t-stat)	(−0.20)		(2.79)	(2.25)		(3.97)
$\beta'\lambda$	2.06	2.05		1.13	1.15	
(t-stat)	(5.25)	(5.47)		(5.49)	(5.73)	
$\bar{\epsilon}$		−0.00	−0.08		0.63	−8.03
(t-stat)		(−0.18)	(−2.68)		(2.32)	(−3.55)
R^2 (%)	0.05	0.05	0.01	0.06	0.06	0.03

slopes remain economically large. For $\bar{r}$, the slope falls from a significant 0.87 in the univariate regression to 0.63 in the bivariate regression. This offers some insight to the source Grundy and Martin's (2001) conclusions: They rely on ad hoc pre-existing factors, and rely moreover on betas estimated from rolling-window regressions. These are sources of model misspecification that assign too much of the interesting structure in returns to their model residuals.

5.3 Another Take On Residual Momentum

In our main analysis, residual momentum is defined based on the error in (2). That is, our main analysis builds residual momentum from the IPCA *forecasting* error,

$$\epsilon_{i,t+1} = r_{i,t+1} - E_t[r_{i,t+1}] = r_{i,t+1} - \beta_{i,t}\lambda_t.$$

By defining the residual in this way, we can decompose stock momentum into the model-based return forecast, and momentum in the forecast residual.

A different, but equally interesting, notion of residual momentum is the use the error relative the the *contemporaneous* factor realization,

$$\tilde{\epsilon}_{i,t+1} = r_{i,t+1} - \beta_{i,t}' f_{t+1}$$

following equation (3). The difference is that $\epsilon_{i,t+1}$ contains shock realization of the factors $(f_{t+1} - \lambda_t)$ while the $\tilde{\epsilon}_{i,t+1}$ strips out the full contemporaneous realization of f_{t+1}. Therefore, momentum in $\tilde{\epsilon}_{i,t+1}$ is a closer analogy to the residual momentum studied by Grundy and Martin (2001).

In Table X, we repeat the analysis of Table III but instead define residual momentum based on the contemporaneous model errors, $\tilde{\epsilon}_{i,t+1}$. The results change none of our conclusions from the main analysis. Residual momentum based on contemporaneous errors is fully explained by the IPCA model. It is in fact weaker than momentum based on prediction errors. This finding is perhaps unsurprising as prediction errors (which include the realized factor innovations) are subject to any momentum that may exist in the factors themselves (e.g., Gupta and Kelly, 2018).

5.4 Time-varying Risk Premia

Up until now we have studied model-based expected returns defined as $E_t[r_{i,t+1}] = \beta_{i,t}\lambda$. That is, we have treated the conditional factor risk premium as constant, $E_t[r_{i,t+1}] = \lambda$. In doing so, we have emphasized the central role of time-varying factor exposures for under-

Table X
IPCA and Contemporaneous Residual Momentum

Notes – See Table III. In this table, residual momentum ($\bar{\epsilon}$ columns) is defined based on contemporaneous IPCA model errors, $\tilde{\epsilon}_{i,t}$, as opposed to prediction errors $\epsilon_{i,t+1}$.

A. Univariate Regressions

	Raw signal			Rank signal		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Constant	0.01	0.00	0.01	0.01	0.01	0.01
(t-stat)	(3.87)	(0.74)	(4.06)	(4.06)	(4.04)	(4.06)
Coeff	0.00	0.86	0.00	0.87	1.92	0.57
(t-stat)	(0.12)	(11.48)	(1.30)	(3.39)	(11.10)	(2.67)
R^2 (%)	0.00	0.13	0.00	0.02	0.12	0.01

B. Portfolio Sorts

	Average return			Sharpe ratio		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Q1	5.97	0.01	7.41	0.23	0.00	0.29
Q2	8.11	6.82	9.47	0.44	0.38	0.52
Q3	9.90	9.88	9.97	0.59	0.54	0.60
Q4	11.87	13.77	10.22	0.69	0.70	0.60
Q5	14.99	20.36	13.77	0.68	0.94	0.64
Q5–Q1	9.02	20.35	6.36	0.53	1.71	0.45
(t-stat)	(3.75)	(12.21)	(3.18)	(3.73)	(11.54)	(3.01)

C. Bivariate Regressions

	Raw signal			Rank signal		
	1	2	3	4	5	6
Constant	0.00	0.00	0.01	0.01	0.01	0.01
(t-stat)	(0.68)	(0.64)	(3.55)	(4.04)	(4.04)	(4.06)
$\bar{r}$	−0.00		−0.01	0.29		2.46
(t-stat)	(−0.73)		(−0.47)	(1.01)		(3.07)
$\beta'\lambda$	0.92	0.90		1.82	1.85	
(t-stat)	(8.75)	(9.85)		(8.68)	(9.61)	
$\bar{\epsilon}$		−0.00	0.01		0.31	−1.71
(t-stat)		(−0.66)	(0.68)		(1.15)	(−2.44)
R^2 (%)	0.14	0.14	0.01	0.12	0.12	0.04

standing the behavior of expected returns. We now investigate whether allowing for time variation in factor risk premia—i.e., allowing for λ_t—changes any of our preceding conclu-

Table XI
IPCA Out-of-sample Performance With Time-varying Lambda

Notes – See Table III. Model-based conditional expected returns use λ_t estimated from a vector autoregression for factor returns, in place of a constant λ.

A. Univariate Regressions

	Raw signal			Rank signal		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Constant	0.01	0.00	0.01	0.01	0.01	0.01
(t-stat)	(3.80)	(1.61)	(3.97)	(3.99)	(3.98)	(3.99)
Coeff	0.00	0.62	0.00	0.87	2.78	0.73
(t-stat)	(0.12)	(4.95)	(0.26)	(3.27)	(10.04)	(2.84)
R^2 (%)	0.00	0.62	0.00	0.02	0.24	0.02

B. Portfolios

	Average return			Sharpe ratio		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Q1	6.34	−3.23	7.05	0.25	−0.15	0.28
Q2	8.54	5.29	9.12	0.46	0.28	0.50
Q3	10.26	10.47	9.79	0.62	0.57	0.59
Q4	12.29	15.45	11.80	0.72	0.82	0.68
Q5	15.06	24.50	14.74	0.68	1.12	0.66
Q5–Q1	8.71	27.73	7.68	0.50	1.49	0.46
(t-stat)	(3.41)	(10.08)	(3.14)	(3.40)	(9.66)	(3.00)

C. Bivariate Regressions

	Raw signal			Rank signal		
	1	2	3	4	5	6
Constant	0.00	0.00	0.01	0.01	0.01	0.01
(t-stat)	(1.60)	(1.60)	(3.40)	(3.98)	(3.98)	(4.00)
$\bar{r}$	−0.00		−0.01	0.44		2.97
(t-stat)	(−0.17)		(−0.69)	(1.63)		(3.56)
$\beta'\lambda$	0.62	0.62		2.71	2.74	
(t-stat)	(4.95)	(4.95)		(9.55)	(9.87)	
$\bar{\epsilon}$		0.00	0.01		0.54	−2.16
(t-stat)		(0.22)	(0.79)		(2.12)	(−2.79)
R^2 (%)	0.62	0.62	0.01	0.25	0.25	0.03

sions and/or enhances the predictive power of IPCA model-based expected returns.

We estimate λ_t as the vector of forecasted factor returns based on a vector autoregression

(VAR). Our VAR takes the form

$$f_{t+1} = \Pi_0 + \Pi_1 f_t + \eta_{t+1} \tag{5}$$

and thus defines λ_t as the out-of-sample fitted value $\Pi_0 + \Pi_1 f_t$.[12] There are some obvious limitations to this approach. For example, (5) is a second-stage regression using generated regressors, and other factor forecasting models (with additional factor predictor variables) may be superior to (5). To this extent, our analysis here should be viewed as an indicative robustness test as opposed to an exhaustive analysis.

Panel A of Table XI describes IPCA out-of-sample model performance versus momentum when using λ_t in place of λ (cf. Table IV). The first finding of the table is that time-varying factor risk prices provide a large boost to IPCA's predictive R^2. The univariate model-based predictive R^2 rises to 0.62% per month with λ_t, compared to 0.09% with static λ. But despite the increase in predictive R^2, accounting for factor risk dynamics has small deleterious effect on the IPCA-based Q5–Q1 Sharpe ratio (1.5 in Table XI versus 1.7 in Table III). And IPCA continues to fully subsume the momentum effect in the bivariate regressions of Panel C column 4.[13] We therefore conclude that the key aspect of IPCA for capturing momentum profits are the dynamics in risk exposures, as opposed to dynamics in risk prices. These conclusions clarify that Chordia and Shivakumar's (2002) results most likely reflect predictable variation in risk exposures instead of risk premia. A varying risk premia does nothing to enhance the profitability of our $\beta'\lambda$ strategy.[14]

5.5 Predicting Realized Betas With IPCA

Section 2 provides strong evidence that characteristics forecast realized betas on Fama-French factors. However, KPS show that the Fama-French model is inferior to IPCA in terms of both explained return covariation and explained differences in average returns (both in-sample and out-of-sample). Furthermore, IPCA prescribes a specific linear combination of characteristics, given by $z'_{i,t}\Gamma$, as the best predictors of stock i's beta on each IPCA factor.

[12] More precisely, we estimate (5) in a recursive out-of-sample manner. That is, given factor estimates $\{f_1, f_2, \ldots, f_t\}$, we estimate the VAR parameters, yielding time-t estimates denoted $\Pi_{0,t}, \Pi_{1,t}$—we abstracted from time subscripts in the main text to ease notation. Hence our estimate of $\lambda_t = E_t(f_{t+1})$ is $\Pi_{0,t} + \Pi_{1,t} f_t$ and f_{t+1} is not used at any point in the model estimation.

[13] Out-of-sample results using the KPS data sample are in Table AV in the appendix, and support the qualitative points made here.

[14] It is worth noting, however, that the stock-level predictions become much more accurate. This opens scope for future work to develop more sophisticated portfolio constructions (than a simple quintile sort) that could take advantage of this accuracy and perhaps find meaningful gains from factor-timing.

In this section, we return to our original motivation of beta prediction and test whether the characteristic combination dictated by IPCA is indeed a superior predictor for exposures to IPCA factors on an out-of-sample basis.

To do so, we first construct daily realizations of our estimated factors. We estimate the model on the monthly primary sample data. We then hold the $\hat{\Gamma}$ estimate fixed and construct out-of-sample daily IPCA factor returns $\hat{f}_{\tau \in t}$ using data on realized stock returns (for each day τ in month t) and characteristics as of month $t-1$. This is out-of-sample because the daily were not used in the estimation of Γ. We then calculate the realized out-of-sample beta of each stock on each IPCA factor using daily returns within month t, which we denote RB_t^{OOS}. To test the predictive power of IPCA betas for out-of-sample realized betas, we regress RB_t^{OOS} on the conditional beta ($\hat{\beta}_{i,t-1} = z_{i,t-1}\hat{\Gamma}$) dictated by IPCA and based on characteristics of month $t-1$, prior to the daily returns used to construct RB_t^{OOS}.

Table XII reports the results. IPCA conditional betas are powerful and unbiased predictors of future realized factor exposures. The IPCA $\beta_{i,t-1}^k$ receives a slope coefficient that is very nearly one in all cases and, while the constant is usually statistically significant, its economic magnitude is tiny. The R^2s are significant and range between 1-2.5% for monthly realized betas, between 3-9% for three-month realized betas, and 5-16% for twelve-month realized betas. Taking account of the sampling error in RB_t^{OOS}, as described in Appendix A3, substantially strengthens these results, with adjusted R^2s ranging between 20-74% for monthly realized betas.[15]

For comparison, Table XIII assesses the predictability in realized IPCA betas based on *lagged realized betas*, rather than using the characteristic-based conditional beta dictated by the model. The results clarify the value of the conditioning information. While lagged realized betas are useful predictors, they are far noisier—essentially the sampling error implicit in calculated realized betas now affects both the dependent and explanatory variable. Thus lagged realized betas are far weaker predictors of realized risk than the betas constructed from a large set of conditioning characteristics.

Table XII
Predicting Realized Betas with IPCA Betas

Notes – Standard errors clustered by month and firm, and there are 2,008,759 observations in each regression. The Adjusted R^2 calculation is described in the appendix.

	Factor				
	1	2	3	4	5
A: One-Month					
$\hat{\beta}^k_{i,t-1}$	1.02	1.02	1.00	1.00	1.01
	(126.99)	(228.83)	(147.57)	(116.70)	(132.88)
Constant	−0.01	−0.01	−0.01	0.01	−0.01
	(−2.56)	(−6.77)	(−4.00)	(3.16)	(−1.76)
R^2 (%)	1.16	2.58	1.77	1.12	0.79
Adjusted R^2 (%)	20.14	43.88	71.37	73.68	41.82
B: Three-Month					
$\hat{\beta}^k_{i,t-1}$	1.05	1.02	1.01	1.01	1.01
	(107.19)	(211.93)	(127.05)	(109.78)	(102.78)
Constant	−0.02	−0.01	−0.01	0.01	−0.01
	(−5.67)	(−5.18)	(−4.55)	(2.64)	(−1.89)
R^2 (%)	4.06	8.70	5.70	3.63	2.74
Adjusted R^2 (%)	24.83	50.85	71.88	71.60	52.39
C: Twelve-Month					
$\hat{\beta}^k_{i,t-1}$	1.15	1.01	1.05	1.03	1.01
	(89.60)	(180.01)	(104.00)	(90.83)	(87.89)
Constant	−0.05	−0.01	−0.01	0.01	−0.02
	(−10.97)	(−2.25)	(−6.49)	(2.55)	(−2.31)
R^2 (%)	7.85	15.77	10.79	5.92	4.63
Adjusted R^2 (%)	18.34	35.10	41.34	33.40	25.14

5.6 Momentum Crashes

Daniel and Moskowitz (2016) investigate momentum "crashes"—periods during which the momentum strategy loses at least half its value. Of the two crashes they focused on, the 2009 crash is in our post-1966 data. Over the three-month period from March to May 2009, the $\bar{r}$ momentum strategy lost a cumulative 51.7%. This is shown in Figure 1, as the dashed

[15] Notice that adjusted R^2s moderately rise for the three-month realized betas, but then fall for the twelve-month. This is the result of two forces at work in the data. The first is an increasing number of observations used to estimate RB_t^{OOS}, which the Monte Carlo exercise captures. The second is the evidence that betas are time-varying in the data and therefore not constant over an increasing horizon, which the Monte Carlo exercise does not capture. Therefore the simulated R^2s in Appendix A3 should be expected to overstate the amount of realized beta predictability at increasing horizons.

Table XIII
Predicting Realized Betas with Realized Betas

Notes – Standard errors clustered by month and firm, and there are 2,008,759 observations in each regression. The Adjusted R^2 calculation is described in the appendix. The monthly IPCA estimation is conducted out-of-sample to construct both realized betas.

	Factor				
	1	2	3	4	5
$RB^k_{i,t-1}$	0.02	0.04	0.02	0.02	0.01
	(7.62)	(12.47)	(10.76)	(9.69)	(8.05)
Constant	0.35	0.49	-0.12	0.11	0.50
	(152.78)	(170.65)	(-38.10)	(32.37)	(107.98)
R^2 (%)	0.03	0.13	0.05	0.03	0.02
Adjusted R^2 (%)	0.55	2.16	1.86	2.09	1.31

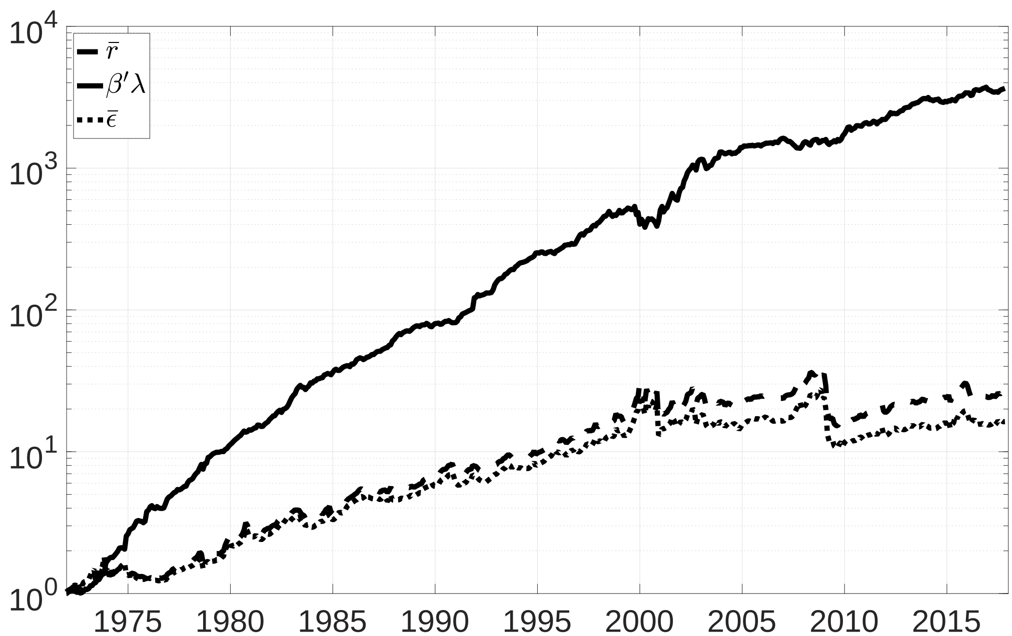

Figure 1: Cumulative Momentum Strategy Returns

Notes – Cumulative strategy returns on log (base ten) scale. The dashed line shows the quintile spread strategy for total return momentum $\bar{r}$, the solid line shows that based on IPCA $\beta'\lambda$, and the dotted line shows residual momentum $\bar{\epsilon}$. The analysis is based on our primary data sample with IPCA estimated out-of-sample.

line falls steeply in the first half of 2009. According to the decomposition in the figure, the crash is due almost entirely to residual momentum. The $\bar{\epsilon}$ strategy (the dotted line) loses a

Table XIV
Strategy Summary Statistics

Notes – Summary statistics from the Q5, Q1, and Q5-Q1 spread portfolios quarterly returns, constructed using return momentum $\bar{r}$, expected returns $\beta'\lambda$, or residual momentum $\bar{\epsilon}$. Numbers are non-annualized. $\beta'\lambda$ comes from out-of-sample IPCA estimated on the primary sample.

	Q5			Q1			Q5-Q1		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Mean	3.93	5.09	3.84	1.76	0.29	1.94	2.21	4.77	1.94
Standard Deviation	12.49	12.38	12.46	14.51	11.17	14.43	8.78	6.71	8.63
Skewness	0.12	0.32	0.08	1.17	0.14	1.20	-1.07	0.86	-1.11
Excess Kurtosis	1.14	1.97	1.11	6.87	2.49	6.88	6.95	3.48	7.24
Minimum	-38.45	-43.62	-39.22	-47.94	-40.69	-47.84	-51.67	-25.18	-50.96
1^{st} Percentile	-30.43	-28.77	-30.26	-32.72	-30.92	-32.28	-29.78	-10.50	-31.37
Median	3.89	4.80	3.81	0.84	0.71	0.98	2.55	4.31	2.26
99^{th} Percentile	35.83	35.51	35.18	48.71	29.49	49.77	23.87	26.36	23.62
Maximum	60.88	69.02	60.14	108.32	57.90	107.80	43.61	38.00	42.72

cumulative 51.0% from March to May 2009, virtually identical to total return momentum. The momentum crash, on the other hand, is essentially absent from the $\beta'\lambda$ strategy. From March to May 2009, the IPCA spread portfolio loses 6.1%.

The largest ever three-month loss for the $\beta'\lambda$ strategy is 25.2% and occurs around the unwinding of the tech boom in early 2000. This and a variety of other summary statistics are reported in Table XIV, using overlapping three-month returns from the underlying monthly strategies.[16] The $\beta'\lambda$ strategy is superior to the $\bar{r}$ momentum strategy according to every risk metric analyzed.

Why does the model-based strategy avoid momentum's fate in 2009? The answer is that IPCA aggregates information across a wide range of conditioning characteristics, only one of which is momentum. Strategies based on other characteristics fared far better than momentum in the first half of 2009. Figure 2 illustrates the heterogeneity in performance across a range long-short quintile spread strategies based on various characteristic that enter the IPCA model. Other return-based characteristics beyond momentum (long-term reversal, short-term reversal, and price relative to 52-week high) also experienced negative returns during the crash. At the same time, size, value, investment, and idiosyncratic volatility performed well. Size returned 30% and idiosyncratic volatility returned 58% over this period.[17] Over the full sample, the momentum characteristic has a cross-sectional correlation

[16] Monthly returns tell the same story—see Table AVI in the appendix.

[17] Size and idiosyncratic volatility are two characteristics that KPS find are significant in the IPCA model

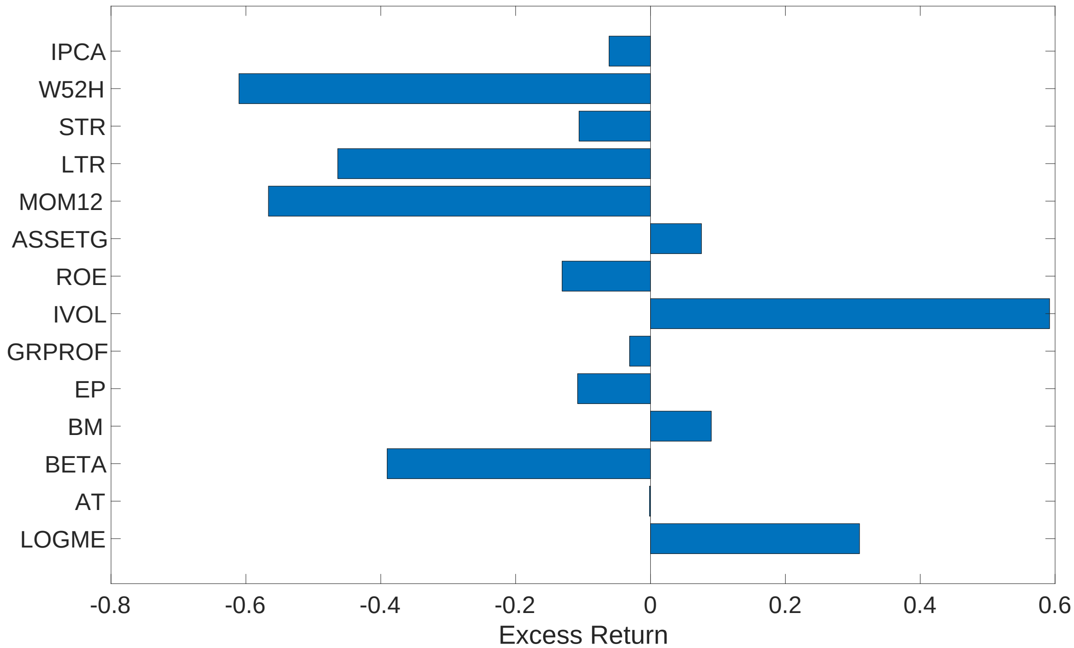

Figure 2: Strategy Returns Over March to May 2009

Notes – Cumulative net excess return for spread (Q5-Q1) portfolios formed by monthly sort based on IPCA $\beta'\lambda$ (top bar), or based on each characteristic individually (remaining bars).

of 0.23 with size and 0.18 with idiosyncratic volatility—but in the six months just before March 2009, the cross-sectional correlation between momentum and size increases to 0.44, and the cross-sectional correlation between momentum and idiosyncratic volatility jumps to 0.55. Hence, the momentum strategy (carrying a positive premium) is hedged attenuated by the size and idiosyncratic volatility signals (which carry negative premia). Heading into the crash, this hedging property become even stronger. Just before the momentum crash, stocks with low momentum are became more likely than usual to be small and have low idiosyncratic volatility. IPCA rolls all of this information into its low dimensional factor structure, as it attempts to identify mean-variance efficient pricing factors. Hence, the IPCA strategy's performance is only mildly affected by the momentum crash because it combines many strategies, some of which performed exceedingly well during the crash, and leverages their mutual hedging behavior.

at the 1% level using KPS data.

Table XV
Out-of-Sample Performance and Turnover

Notes – The turnover ratio is defined as turnover of the strategy divided by turnover of momentum with the same weighting scheme. Average returns and Sharpe ratios are annualized, and returns are in percent.

Sample	Equal-Weight			Value-Weight		
	Turnover Ratio	Average Return	Sharpe Ratio	Turnover Ratio	Average Return	Sharpe Ratio
A. Momentum						
Primary	1.0	8.7	0.5	1.0	10.8	0.5
KPS	1.0	7.3	0.4	1.0	10.6	0.5
B. IPCA						
Primary	2.4	18.7	1.5	2.1	11.3	0.8
KPS	3.0	30.9	2.3	2.3	13.4	0.9
C. IPCA, no return-based characteristics						
Primary	0.2	11.7	0.8	0.2	7.5	0.5
KPS	0.9	14.9	0.9	0.9	10.2	0.6

5.7 Value Weighting and Turnover

Finally, we evaluate two additional dimensions of our results: the impact of value weighting and the turnover involved with forming these portfolios. Until now, our quintile portfolios have been equal-weighted, making them a portfolio analogue to regression results which also equally-weight every stock-month observation. Now we compare equal and value weights in terms of performance and turnover. To make the turnover comparison easier to read, we express turnover relative to the turnover of the basic momentum strategy. A turnover ratio above one means that a strategy has more turnover than the momentum strategy with the same weighting scheme. In this section we only report out-of-sample comparisons.

Table XV shows that the large outperformance of IPCA model's $\beta'\lambda$ strategy (Panel B) relative to momentum (Panel A) is also accompanied by much 2–3 times as much turnover as momentum. And while value weighting has negligible effect on momentum's Sharpe ratio, it reduces IPCA's Sharpe ratio by about half. That said, the value-weighted $\beta'\lambda$ portfolio still produces a Sharpe ratio of 0.8–0.9, greater than momentum's Sharpe ratio of 0.5.

IPCA's portfolio weights change due to the continually updating information in dynamic stock characteristics. The fastest moving characteristics are those based on recent price trends. In Panel C, we examine how IPCA's performance and turnover are affected by eliminating all return-based characteristics from the model. Panel C of Table XV shows that turnover is dramatically reduced by excluding return-based conditioning variables. In the primary sample, turnover drops to one-fifth that of momentum, but still achieves Sharpe ratios of 0.8 and 0.5 on an equal-weighted and value-weighted basis, respectively. In the KPS sample, the turnover is roughly 90% that of momentum, and achieves respective Sharpe ratios of 0.9 and 0.6. The basic conclusion from this analysis is that the high turnover of IPCA-based strategies can be greatly reduced with careful selection of conditioning variables while still managing to outperform the simple momentum strategy.

6 Conclusion

Momentum and reversal are well known predictors of future market returns. We show that these characteristics are strongly predictive of a stock's realized exposures to common risk factors. This fact is direct evidence that these price trend strategies are in part explainable as compensation for risk exposure.

Motivated by this observation, we examine the role of price trends as conditioning information in a full-fledged conditional factor pricing model. To do so, we use an empirical procedure called instrumented principal component analysis (IPCA) that estimates latent risk factors and assets' time-varying betas on these factors.

We show that the momentum and long-term reversal effects are in large part explained by a conditional betas in no-arbitrage factor pricing model. In our main specifications, model-based conditional expected returns supplant momentum and long-term reversal signals as return predictors. This is true in various data samples with different numbers of overall observations and differing richness in the available characteristics. Spread portfolios formed on the basis are model-implied conditional expectations have annualized Sharpe ratios more than double that of the the momentum strategy. These conclusions hold both in-sample and out-of-sample, for equal-weighted and value-weighted strategies, after accounting for differences in turnover, and historically avoided large drawdowns such as the momentum crash of early 2009.

References

Asness, C., A. Frazzini, R. Israel, and T. Moskowitz (2014): "Fact, Fiction, and Momentum Investing," *The Journal of Portfolio Management*, 40(5), 75–92.

Asness, C. S. (1994): "Variables That Explain Stock Returns," *dissertation, University of Chicago*, 00000.

Boguth, O., M. Carlson, A. Fisher, and M. Simutin (2011): "Conditional risk and performance evaluation: Volatility timing, overconditioning, and new estimates of momentum alphas," *Journal of Financial Economics*, 102(2), 363–389.

Bondt, W. F. M. D., and R. Thaler (1985): "Does the Stock Market Overreact?," *The Journal of Finance*, 40(3), 793–805.

Carhart, M. M. (1997): "On Persistence in Mutual Fund Performance," *The Journal of Finance*, 52(1), 57–82.

Chordia, T., and L. Shivakumar (2002): "Momentum, Business Cycle, and Time-varying Expected Returns," *The Journal of Finance*, 57(2), 985–1019, 01096.

Conrad, J., and G. Kaul (1998): "An Anatomy of Trading Strategies," *The Review of Financial Studies*, 11(3), 489–519.

Daniel, K., and T. J. Moskowitz (2016): "Momentum crashes," *Journal of Financial Economics*, 122(2), 221–247.

Ehsani, S., and J. T. Linnaimaa (2019): "Factor Momentum and the Momentum Factor," Discussion Paper 25551, NBER.

Fama, E. F., and K. R. French (2015): "A five-factor asset pricing model," *Journal of Financial Economics*, 116(1), 1–22.

——— (2016): "Dissecting Anomalies with a Five-Factor Model," *The Review of Financial Studies*, 29(1), 69–103, 00315.

Grundy, B. D., and J. S. M. Martin (2001): "Understanding the Nature of the Risks and the Source of the Rewards to Momentum Investing," *The Review of Financial Studies*, 14(1), 29–78, 01043.

Hong, H., and J. C. Stein (1999): "A Unified Theory of Underreaction, Momentum Trading, and Overreaction in Asset Markets," *The Journal of Finance*, 54(6), 2143–2184.

Jegadeesh, N. (1990): "Evidence of Predictable Behavior of Security Returns," *The Journal of Finance*, 45(3), 881–898.

Jegadeesh, N., and S. Titman (1993): "Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency," *The Journal of Finance*, 48(1), 65–91.

——— (2002): "Cross-Sectional and Time-Series Determinants of Momentum Returns," *The Review of Financial Studies*, 15(1), 143–157.

Kelly, B. T., S. Pruitt, and Y. Su (forthcoming): "Characteristics Are Covariances: A Unified Model of Risk and Return," *Journal of Financial Economics*.

Lewellen, J., and S. Nagel (2006): "The conditional CAPM does not explain asset-pricing anomalies," *Journal of Financial Economics*, 82(2), 289–314.

Liu, L. X., and L. Zhang (2014): "A neoclassical interpretation of momentum," *Journal of Monetary Economics*, 67, 109–128.

Lo, A. W. (2002): "The Statistics of Sharpe Ratios," *Financial Analysts Journal*, 58(4), 36–52.

Vayanos, D., and P. Woolley (2013): "An Institutional Theory of Momentum and Reversal," *The Review of Financial Studies*, 26(5), 1087–1145.

Appendix

A1 Long-term Reversal Simulations

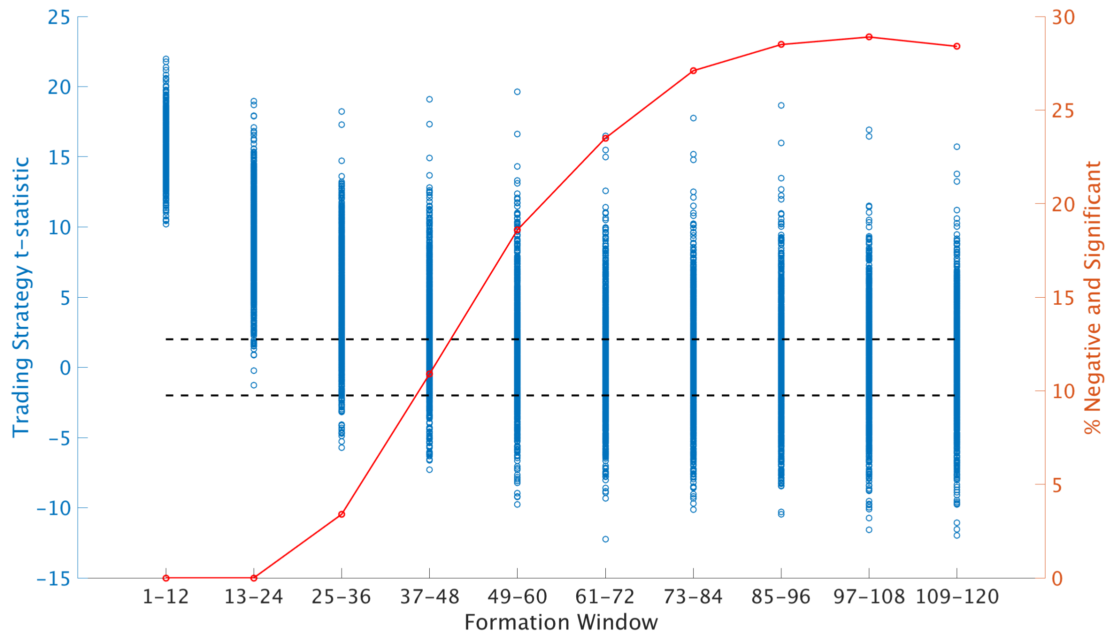

Figure A1: Simulated Momentum/Reversal Strategies

Notes – Blue circles show the distribution of t-statistics for a momentum strategy with various formation windows. The red line shows the fraction of simulations in which a reversal strategy is statistically significant.

The following example illustrates via simulation how a long-term reversal strategy has an unusually high propensity to falsely generate significant trading strategy performance when the true return data generating has simple, positive AR(1) dynamics in expected returns.

The simulation model has the following simple structure. Expected returns of all assets follow a persistent AR(1) and are driven by a common expected return shock plus an iid idiosyncratic component:

$$\mu_{i,t} = 0.95\mu_{i,t-1} + 0.1u_{i,t}, \quad u_{i,t} = \beta_i u_{0,t} + 0.1\nu_{i,t}$$

where $u_{0,t}$ and $\nu_{i,t}$ are iid standard normals and β_i is uniform [-0.5,0.5]. Return realizations are given by

$$r_{i,t+1} = \mu_{i,t} + 10\epsilon_{i,t}$$

where $\epsilon_{i,t}$ is iid standard normal, thus returns have a predictive R^2 of roughly 1% per month.

In each of 1,000 simulated samples, we generate a panel of 5,000 stock returns over 500 months. We then sort stocks into deciles based on their average returns over various formation windows, including past 1-12 months, 13-24 months, ... , 109-120. For each decile sort, we computer the t-statistic for average returns on the 10-1 portfolio spread. Because the data is generate by a simple and highly persistent AR(1) model, we expect that average returns over short and intermediate past horizons will be predictive of future returns and tend to generate significantly positive 10-1 trading strategy returns. At long horizons, we expect this predictability to die out and become insignificant.

The results are shown in Figure A1. Indeed, we find that with a 1-12 month formation window, the momentum strategy's performance is positive and significant in every simulated sample. Perhaps surprisingly, as look back window lengthens, the number of significantly negative (i.e., *reversal*) strategies proliferates. Looking back five years (49-60 months), long-term reversal is significant in about 20% of simulations, and reaches as high as 30% for the seven-year look-back.

In summary, it is in no way unusual to detect significant long-term reversals in data with strictly positively persistent expected return dynamics.

A2 Daily IPCA Factor Construction

Estimating IPCA at the monthly frequency expresses the idea that betas may vary from month-to-month. When we construct daily factor realizations, we use the Γ estimated from monthly-frequency characteristics and returns, along with those monthly-frequency characteristics. This embodies the assumption that betas may change between months, but are constant within each month.

Recall (cf. KPS) that the IPCA factor estimate in month t is given by a cross-sectional regression of stock returns realized in month t on betas realized in month $t-1$:

$$f_t = \left(\beta_{t-1}'\beta_{t-1}\right)^{-1}\beta_{t-1}'r_t$$

where β_{t-1} is the $(N \times K)$ matrix of the K factor exposures for each of the N stocks, whose returns are in the $N-$vector r_t.[18] The IPCA model assumes that

$$\beta_{t-1} = Z_{t-1}\Gamma$$

for a $(N \times L)$ matrix of observable characteristics Z_{t-1} that are mapped to risk exposures via the asset- and time-invariant mapping Γ. Therefore the factor estimate can be written

$$\begin{aligned} f_t &= \left(\Gamma' Z_{t-1}' Z_{t-1} \Gamma\right)^{-1} \Gamma' Z_{t-1}' r_t \\ &= \left(\Gamma' Z_{t-1}' Z_{t-1} \Gamma\right)^{-1} \Gamma' x_t \end{aligned}$$

where x_t is the $L-$vector of characteristic-managed portfolio realizations given by the cross product of the month-$(t-1)$ characteristics with the month-t stock returns.

To create instead daily factor realizations *within* month t, all we must do is change the characteristic-managed portfolio realizations to be at the daily frequency. Let $\tau(t)$ be a day within month t and $r_{\tau(t)}$ the $N-$vector of stock returns realized that day. Then

$$x_{\tau(t)} = Z_{t-1}' r_{\tau(t)}$$

are the returns on the L characteristic-managed portfolios on day τ of month t. The daily factor estimate is then the linear combination of these portfolios' daily returns given by

$$f_{\tau(t)} = \left(\Gamma' Z_{t-1}' Z_{t-1} \Gamma\right)^{-1} \Gamma' x_{\tau(t)}.$$

A3 Monte Carlo Approach to Adjusting the Predictive R^2s

A fundamental statistical issue with predicting realized betas RB_t^{OOS} using the IPCA betas β_{t-1} (subsuming the dependence on i for simplicity) is that the former is estimated using a small number of observations—the number of days in the month. Hence the dependent variable RB_t^{OOS} is measured with error. This drives down the predictive R^2 of the regression of RB_t^{OOS} on β_{t-1}, even when the null model is data generating process. To illustrate this

[18] This is a common expression. If betas were static, this is also the factor estimate of (asymptotic) principal components in Chamberlain and Rothschild (1983) and Connor and Korajcyzk (1986). Furthermore, to the extent that portfolio sorts can also be cast as a regression on indicator-like regressors, this is also how sorted portfolios a la Fama and French (1993) are constructed (although in multiple univariate regressions instead of a multivariate regression).

effect and get a sense of its magnitude, we conduct the following Monte Carlo exercise. It essentially imagines that we have increasing number of "days" in each month with which to estimate RB_t^{OOS} and shows how this increasing precision would lead the predictive regression of RB_t^{OOS} on β_{t-1} to R^2s of 100%.

Take the β_{t-1} as given. Generate D "daily" factor innovations as random normals parameterized by the monthly estimates of the factors' means and standard deviations, translated to the actual daily frequency.[19] Generate D "daily" idiosyncratic returns for each stock as random normals parameterized by their monthly standard deviations, translated to the actual daily frequency. Construct D "daily" returns using (3). We then use these generated "daily" returns as if they were the actual daily data, and estimate D "daily" factor realizations. From these D "daily" factor realizations we estimate a realized beta RB_t^{OOS} for every month. If $D = 22$ then we have essentially randomly generated as many "daily" returns as there are in the actual daily data. But if we let D increase, we are randomly generating more "days" in every month, which intuitively will allow every month's estimate of RB_t^{OOS} to become more precise. Finally, we run the predictive regression of the simulated RB_t^{OOS} on the exact same β_{t-1} that our simulation used to create the stock returns according to (3).

Table AI Panel A shows the predictive R^2 that come from these simulated regressions, for differing values of D. When $D = 22$, as is about the case in the actual daily data, these predictive R^2s are low. They range between 1.5 to 5.9%. But as D rises, these R^2s rise, too.[20] If we had 20000 "daily" observations, the predictive R^2s would all be higher than 94%. Hence, even when the null model is true we see diminished predictive R^2s because there are only a few daily observations available to estimate RB_t^{OOS}. Were we able to make the estimates of RB_t^{OOS} more and more precise, the predictive R^2 appear to converge to 100%.

This simulation evidence suggests that sampling error contaminates our RB_t^{OOS} estimates, much as Lewellen and Nagel (2006) pointed out. Therefore, we provide adjusted R^2 measures that take this sampling error into account. For Table XII we simply divide the actual R^2 by the values in the corresponding rows of Table AI ($D = 22$), respectively: the 1-month us $D = 22$, the 3-month use $D = 66$, and the 12-month use $D = 250$. For Table II, which use Fama and French (2015) factors that do not correspond to our IPCA-estimated factors, we divide the actual R^2s by the average R^2 from the corresponding rows of Table AI Panel A.

[19] For example, if the monthly factor mean is μ (expressed as a rate), then we find the daily factor mean as $(1 + \mu)^{1/22} - 1)$.

[20] We stop at $D = 20000$ because it takes approximately 32GB of RAM for Matlab to work with the simulated "daily" data, and moreover by this time our point has been made.

Table AI
Simulated Realized Beta Predictability

Notes – Predictive R^2 (%) from the predictive regression of RB_t^{OOS} on β_{t-1} in simulated data. The simulated data use D "daily" observations to estimate RB_t^{OOS} each month. There are always 2,008,759 stock-month observations for each predictive regression, just as in the real data. Panel A uses the IPCA β to predict, while Panel B uses only the momentum characteristic to predict.

D	Factor				
	1	2	3	4	5
A. IPCA β					
22	5.76	5.88	2.48	1.52	1.65
66	16.35	17.11	7.93	5.07	5.23
250	43.80	44.94	26.10	17.42	18.42
500	60.85	61.86	41.04	29.67	31.21
2000	86.23	86.67	73.68	63.00	64.42
10000	96.90	97.03	93.30	89.44	90.14
20000	98.42	98.49	96.56	94.41	94.79
B. Momentum					
22	1.65	0.00	0.20	0.39	0.44
66	4.89	0.00	0.59	1.29	1.37
250	12.90	0.01	2.13	4.19	4.72
500	18.04	0.01	3.41	7.28	7.93
2000	25.41	0.01	5.98	15.45	16.61
10000	28.58	0.01	7.47	22.01	23.29
20000	29.11	0.01	7.86	23.15	24.47

Panel B of Table AI shows the predictive R^2 that come from these simulation regressions, for differing values of D, when we predict the future realized beta RB_t^{OOS} using only momentum. Since the premise of this paper is that multiple characteristics carry information about stock exposures, it should come as no surprise that restricting the predictive information only to momentum drops the predictive accuracy. We have no expectation that these R^2s would converge to 100% as D increases: momentum is only one signal among many that tell us how stock returns will covary with aggregate risks. According to the IPCA model β is the correct linear combination of characteristics to understand future aggregate covariance, and any one characteristic will be insufficient for the task, even as the dependent variable RB_t^{OOS} is measured with arbitrary accuracy. For Table I, which use Fama and French (2015) factors that do not correspond to our IPCA-estimated factors, we divide the actual R^2s by the average R^2 form the corresponding rows of Table AI Panel B.

A4 Additional Tables

Table AII
Predicting Realized Betas With Characteristics (Univariate Regressions)

Notes – Standard errors clustered by month and firm.

	Factor				
	MKTRF	SMB	HML	RMW	CMA
LOGME	0.73	−0.25	0.04	0.18	−0.14
	(36.42)	(−8.31)	(1.34)	(5.34)	(−4.18)
R^2 (%)	0.73	0.04	0.00	0.01	0.01
AT	0.54	−0.39	0.42	0.35	−0.10
	(27.01)	(−13.52)	(14.95)	(10.95)	(−2.91)
R^2 (%)	0.35	0.10	0.06	0.03	0.00
BETA	1.05	0.61	−0.18	−0.44	−0.10
	(70.70)	(27.31)	(−6.00)	(−14.60)	(−3.06)
R^2 (%)	1.43	0.25	0.01	0.05	0.00
BM	−0.34	−0.11	0.69	0.23	0.11
	(−25.33)	(−6.03)	(33.46)	(9.82)	(5.33)
R^2 (%)	0.15	0.01	0.16	0.01	0.00
EP	−0.07	−0.19	0.51	0.67	−0.22
	(−4.43)	(−10.09)	(20.51)	(23.59)	(−7.57)
R^2 (%)	0.01	0.02	0.09	0.12	0.01
GRPROF	0.09	0.16	−0.26	0.31	−0.01
	(5.94)	(8.19)	(−11.89)	(11.42)	(−0.25)
R^2 (%)	0.01	0.02	0.02	0.02	0.00
IVOL	0.10	0.66	−0.35	−0.63	0.06
	(4.01)	(23.85)	(−10.41)	(−16.78)	(1.55)
R^2 (%)	0.01	0.29	0.04	0.10	0.00
ROE	0.15	−0.19	0.16	0.69	−0.33
	(10.08)	(−8.97)	(5.95)	(22.75)	(−10.92)
R^2 (%)	0.03	0.02	0.01	0.11	0.02
ASSETG	0.18	0.06	−0.04	0.04	−0.60
	(17.43)	(4.28)	(−2.48)	(2.00)	(−27.24)
R^2 (%)	0.04	0.00	0.00	0.00	0.08
LTR	0.27	0.13	−0.28	−0.00	−0.23
	(16.70)	(5.79)	(−9.80)	(−0.15)	(−7.23)
R^2 (%)	0.10	0.01	0.03	0.00	0.01
STR	0.05	−0.06	−0.01	0.11	−0.01
	(2.61)	(−2.51)	(−0.39)	(3.56)	(−0.33)
R^2 (%)	0.00	0.00	0.00	0.00	0.00
W52H	−0.07	−0.38	0.13	0.47	−0.01
	(−3.52)	(−13.59)	(3.36)	(11.71)	(−0.30)
R^2 (%)	0.01	0.10	0.01	0.06	0.00

Table AIII
Characteristics

Notes – Solid bullets indicate the characteristics in our benchmark sample. The complete list are the characteristics in the KPS sample. The return-based characteristics dropped in Table XV are Momentum, Long-term reversal, Short-term reversal, Price rel. to 52-week high, and Intermediate momentum (for the KPS sample results).

- ● Log market equity
- ● Assets
- ● Market beta
- ● Book to market
- ● Earnings to price
- ● Gross profitability
- ● CAPM idio. vol.
- ● Return on equity
- ● Asset growth
- ● Momentum
- ● Long-term reversal
- ● Short-term reversal
- ● Price rel. to 52-week high
- ○ Tobin's Q
- ○ Assets to market equity
- ○ Net sales to operating assets
- ○ Cash to total assets
- ○ Capital turnover
- ○ Capital intensity
- ○ PPE and inventory over assets
- ○ Fixed costs to sales
- ○ Cash flow to book equity
- ○ Leverage
- ○ Turnover
- ○ Net operating assets
- ○ Operating accruals
- ○ Operating leverage
- ○ Price to cost margin
- ○ Profit margin
- ○ Return on net assets
- ○ Return on assets
- ○ Intermediate momentum
- ○ Sales to market equity
- ○ Sales expenses to sales
- ○ Bid-ask spread
- ○ Standard unexplained volume

Table AIV
IPCA Out-of-sample KPS

Notes – The table uses the KPS data sample. Model-based conditional expected returns are from recursive out-of-sample estimation. See Table III for further table description.

A. Univariate Regressions

	Raw signal			Rank signal		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Constant	0.01	0.00	0.01	0.01	0.01	0.01
(t-stat)	(3.73)	(1.42)	(3.81)	(3.82)	(3.82)	(3.82)
Coeff	−0.00	0.77	−0.00	0.69	3.03	0.61
(t-stat)	(−0.49)	(12.15)	(−0.49)	(2.37)	(13.46)	(2.08)
R^2 (%)	0.00	0.28	0.00	0.01	0.28	0.01

B. Portfolio Sorts

	Average return			Sharpe ratio		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Q1	9.24	−2.89	9.44	0.34	−0.14	0.35
Q2	9.19	7.02	10.02	0.46	0.37	0.51
Q3	11.12	10.84	10.76	0.61	0.56	0.58
Q4	13.35	16.48	12.83	0.72	0.79	0.68
Q5	16.55	27.99	16.39	0.71	1.13	0.71
Q5–Q1	7.31	30.88	6.95	0.42	2.29	0.40
(t-stat)	(2.79)	(15.17)	(2.65)	(2.78)	(13.75)	(2.41)

C. Bivariate Regressions

	Raw signal			Rank signal		
	1	2	3	4	5	6
Constant	0.00	0.00	0.01	0.01	0.01	0.01
(t-stat)	(1.45)	(1.32)	(3.29)	(3.82)	(3.82)	(3.82)
$\bar{r}$	−0.01		−0.00	−0.08		2.82
(t-stat)	(−1.56)		(−0.13)	(−0.24)		(3.50)
$\beta'\lambda$	0.84	0.82		3.05	3.02	
(t-stat)	(11.26)	(11.62)		(11.60)	(12.08)	
$\bar{\epsilon}$		−0.01	−0.00		0.03	−2.16
(t-stat)		(−1.36)	(−0.01)		(0.08)	(−2.72)
R^2 (%)	0.32	0.31	0.00	0.28	0.28	0.02

Table AV
IPCA With Time-varying Lambda, KPS Sample

Notes – This table uses the KPS data sample. Model-based conditional expected returns use λ_t estimated from a vector autoregression for factor returns, in place of a constant λ. See Table III for further table description.

A. Univariate Regressions

	Raw signal			Rank signal		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Constant	0.01	0.00	0.01	0.01	0.01	0.01
(t-stat)	(3.73)	(1.38)	(3.81)	(3.82)	(3.81)	(3.82)
Coeff	−0.00	0.73	−0.00	0.69	4.17	0.54
(t-stat)	(−0.49)	(5.46)	(−0.56)	(2.37)	(14.49)	(1.94)
R^2 (%)	0.00	0.98	0.00	0.01	0.53	0.01

B. Portfolios

	Average return			Sharpe ratio		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Q1	9.24	−8.06	10.15	0.34	−0.37	0.38
Q2	9.19	4.80	9.68	0.46	0.24	0.49
Q3	11.12	10.68	10.87	0.61	0.53	0.59
Q4	13.35	18.80	12.61	0.72	0.93	0.67
Q5	16.55	33.25	16.14	0.71	1.37	0.70
Q5–Q1	7.31	41.32	6.00	0.42	2.45	0.36
(t-stat)	(2.79)	(16.19)	(2.40)	(2.78)	(14.50)	(2.15)

C. Bivariate Regressions

	Raw signal			Rank signal		
	1	2	3	4	5	6
Constant	0.00	0.00	0.01	0.01	0.01	0.01
(t-stat)	(1.49)	(1.38)	(2.92)	(3.81)	(3.81)	(3.82)
$\bar{r}$	−0.00		0.00	−0.35		3.38
(t-stat)	(−1.10)		(0.15)	(−1.15)		(3.10)
$\beta'\lambda$	0.74	0.74		4.26	4.22	
(t-stat)	(5.45)	(5.45)		(13.81)	(14.07)	
$\bar{\epsilon}$		−0.00	−0.01		−0.26	−2.75
(t-stat)		(−1.10)	(−0.26)		(−0.90)	(−2.68)
R^2 (%)	1.00	1.00	0.00	0.53	0.53	0.03

Table AVI
Momentum Strategy Summary Statistics, Monthly Returns

Notes – Summary statistics for the Q5, Q1, and Q5-Q1 spread portfolios monthly returns, constructed using return momentum $\bar{r}$, expected returns $\beta'\lambda$, or residual momentum $\bar{\epsilon}$. Numbers are non-annualized. $\beta'\lambda$ comes from out-of-sample IPCA estimated on the primary data sample.

	Q5			Q1			Q5-Q1		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Monthly Returns									
Mean	1.11	1.44	1.09	0.47	0.06	0.52	0.64	1.38	0.57
Standard Deviation	6.01	5.90	6.02	6.98	5.60	6.90	4.70	3.33	4.69
Interdecile Range	13.57	12.61	13.88	14.78	12.36	14.36	8.81	6.52	8.39
Skewness	-0.37	0.40	-0.40	0.98	-0.32	1.05	-1.65	1.15	-1.71
Kelley Skewness	0.06	0.15	0.07	0.07	-0.01	0.07	0.00	0.16	-0.03
Excess Kurtosis	3.99	7.01	3.84	7.09	3.41	7.48	14.71	9.77	15.64
Minimum	-31.40	-29.12	-31.44	-27.03	-29.71	-25.83	-37.73	-17.43	-38.61
1^{st} Percentile	-16.80	-16.86	-16.63	-16.41	-18.07	-15.99	-15.20	-6.88	-15.70
Median	0.59	0.81	0.53	0.00	0.00	0.00	0.66	1.00	0.58
99^{th} Percentile	14.88	16.14	15.00	22.40	15.53	21.93	11.92	12.85	11.95
Maximum	29.60	42.13	30.08	46.16	25.00	46.38	26.94	23.80	27.04