Better Monitoring . . . Worse Productivity?

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Abstract

I study how the informativeness of monitoring within a firm affects the productivity of the firm's workers. When there is repeated moral hazard and the firm privately monitors worker effort, increasing the information content of monitoring at each date can be counterproductive. Adding new information that is weak in statistical power but strong in incentive power is particularly dangerous, and in some cases can cause the optimal dynamic contract to collapse into providing zero effort incentives. Delaying the firm's ability to react to the stream of information generated by monitoring only exacerbates the problem.

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1 Introduction

Monitoring is integral to economic relationships, generating the information needed to evaluate performance. Inside a firm, monitoring underpins the entire incentive system, ensuring hard work can be rewarded and talent can be recognized and retained.

Recent technological advances have introduced a plethora of new, cost-effective ways for firms to monitor their workers: Humanyze processes vocal information to determine if someone is dominating conversation. OccupEye produces heat sensors that tell when an employee is at a desk. Even dairy cows are being outfitted with wearable technology from Cowlar to help with milk production. Underlying the adoption of such technologies is the assumption that better monitoring equals better production.

Nevertheless ample anecdotal evidence suggests there is such a thing as too much monitoring, which can demoralize workers (at least the human ones), leading to lower productivity.

In this paper, I uncover an *incentives channel* through which better monitoring can hurt productivity and firm value. My better monitoring/worse outcome result does not hinge on monitoring being costly or workers having a psychological reaction to monitoring, although both are important facets of the firm-worker relationship. What my result does hinge on is private monitoring.

For many workers, monitoring is naturally private. Some data about worker performance, such as those being generated by many of the new technologies, are quite personal, and for privacy reasons can only be observed by the firm. Moreover, most workers do not, individually, meaningful impact a firm's public monitoring signals such as stock price or cash flow. Instead, the employer's private opinion of worker performance is much more informative. I model the private monitoring relationship between firm and worker using a principal-agent framework. The agent-worker repeatedly exerts hidden effort and the principal-firm privately observes the information generated by monitoring. Over time, the principal reports on agent performance, affecting the agent's payoff through an optimal dynamic contract specifying pay and termination as a function of the history of reports.

When monitoring is private, the link between agent performance and agent payoff is filtered through the principal's strategic reports. To maximize surplus, the principal should report in a way that fully leverages the *statistical power* of information – for example avoiding messages that lead to very inefficient punishment outcomes unless rare, very negative information is observed. However, the principal is interested in his own utility not surplus and will therefore report in a way that maximizes the *incentive power* of information which generates the most effort from the worker.

My paper's main insight is that maximizing the incentive power of information often means not fully leveraging its statistical power. Improving monitoring changes how the principal reports to maximize incentive power which means that even though statistical power increases, the principal can end up leveraging less of it – so much less in some cases that firm value can actually decline. This is the better monitoring/worse outcome result.

For example, when the information generated by monitoring each date consists of a binary valued "bad news" Poisson increment and a binary valued random walk increment fully leveraging the statistical power of information to maximize surplus involves punishing the agent only when both signals are negative. However, maximizing incentive power involves punishing the agent when either signal is negative. In the continuous-time limit, a random walk increment is very weak statistically, and maximizing incentive power which entails punishing the agent whenever the statistically weak random walk goes down is extremely inefficient. The only way to prevent this inefficiency is to have a trivial contract that never allows the principal to punish. Thus, when a bad news Poisson monitoring technology is improved by adding a Brownian component, the optimal continuous-time contract, which previously induced positive effort, collapses into a trivial contract that pays a flat wage and never terminates the agent. Effort drops to zero and the firm is worse off.

Emphasizing the incentive power of information is the key way in which my model of intra-firm monitoring differs from classic repeated game models of inter-firm monitoring that emphasize statistical power. See, for example, Abreu, Milgrom and Pearce (1991) and Sannikov and Skrzypacz (2007). Two important findings of that literature that both work through statistical power are:

1. Holding the frequency of monitoring fixed, increasing its information content never hurts firm value.
2. Holding information content fixed, reducing frequency often increases firm value.

In contrast, my model of intra-firm monitoring, with its emphasis on incentive power, generates almost opposing comparative statics:

1. Holding frequency fixed, increasing information content often hurts firm value.
2. Holding information content fixed, reducing frequency never increases firm value.

In the last part of the paper, I consider a canonical continuous-time limit model where monitoring is sampling the current value of a fixed stochastic process tracking cumulative productivity. In this setting, the frequency and information content of monitoring are linked. My better monitoring/worse outcome result suggests that limiting the frequency of monitoring can – by limiting information content – be valuable. I then show that optimal monitoring occurs on evenly spaced, nonrandom dates, resulting in positive but cyclic worker productivity.

My paper contributes to the literature on optimal contracting under private monitoring and, more generally, to the repeated games under private monitoring literature. My insight that with private monitoring the principal always maximizes the incentive power of information is inspired by an intuition of Dewatripont (1987) for selecting perfect equilibria in sequential models of spatial competition. This significant connection is explained in detail in Section 2.2. Formally, I adapt the idea of Dewatripont (1987) to refine sequential equilibrium in my model. In addition to featuring a principal that maximizes incentive power, all sequential equilibria that survive my refinement generate public continuation payoff processes and are belief-free. Ely, Horner and Olszewski (2005) study belief-free equilibria in repeated games with private monitoring and highlight their attractive properties. Imposing the refinement significantly alters the optimal contracting problem compared to those that allow all sequential equilibria such as the one considered in Fuchs (2007). In earlier work, Zhu (2018) develops a special case of the refinement and uses it to study when multilateral contracts can be decomposed into a collection of single-agent contracts.

While I show how giving the principal too much information and incentive power can be counterproductive, there is a complementary literature on intrinsic motivation that shows how giving the agent too much information and incentives can be counterproductive. The basic idea there involves an informed principal trying to keep the agent in the dark about his possibly low type. See, for example, Benabou and Tirole (2003) and Orlov (2018).

Cremer (1995), Burkart, Gromb and Panunzi (1997), and Aghion and Tirole (1997) also consider principal-agent models in which there is value to limiting what the principal knows. In Cremer (1995), keeping the principal at arm's length helps the principal commit to not renegotiate when something bad happens – information makes it hard for the principal to be tough. In contrast, information in my model makes it hard for the principal not to be too tough. In Burkart, Gromb and Panunzi (1997), and Aghion and Tirole (1997) contractual incompleteness causes a hold-up problem to arise when the principal has too much information, and monitoring is over the information the agent knows rather than the action the agent takes.

In the last part of my paper I endogenize the monitoring structure of contracts and show that it is optimal not to monitor at all times even though monitoring is free. Piskorski and Westerfield (2016) also endogenize the monitoring structure in a dynamic moral hazard model with public monitoring. Monitoring is costly, and the optimal contract, which takes those costs into account, features infrequent monitoring.

2 Contracts Under Private Monitoring

This section introduces the model and characterizes optimal contracts holding monitoring structure fixed. Sections 3 and 4 then explore how changes to the information content and the frequency of monitoring affect productivity and firm value.

2.1 The Model

The timeframe, $[0, T]$, is split into dates of length Δ, denoted by $t = 0, \Delta, \ldots, T$. The discount factor is $e^{-r\Delta}$ for some $r > 0$. At the beginning of each date t, the principal P pays the agent A some amount $w_t \in \mathbb{R}$. Next, A observes a private randomizing device taking finitely many values and chooses effort $a_t \in [0, 1)$. a_t costs $h(a_t)\Delta$ with $h(0) = h'(0) = 0$, $h'' > 0$, and $\lim_{a_t \to 1} h(a_t) = \infty$. a_t determines the distribution of a signal dX_t taking finitely many values. Given dX_t, P's utility is $u(dX_t)$ with $\mathbf{E}_{a_t} u(dX_t)$ weakly concave in effort and $\mathbf{E}_0 u(dX_t) > 0$.

After A exerts effort, P *monitors* A: First, P observes a private randomizing device taking finitely many values and dX_t; then, P reports a public message m_t selected from a contractually pre-specified finite set of messages $\mathcal{M}$; next, a public randomizing device is realized; finally, A is randomly terminated at the beginning of date $t + \Delta$. If A is terminated A and P exercise outside options worth 0 at date $t + \Delta$ and P makes a final payment $w_{t+\Delta}$ to A. A is always terminated at date T.

A *contract game* $(\mathcal{M}, w, \tau)$ specifies a finite message space $\mathcal{M}$, a payment plan w, and a termination clause τ. Let h_t denote the public history of messages and public randomizing devices up to the end of date t. w consists of an $h_{t-\Delta}$-measurable payment w_t to the agent for each t. τ is a stopping time where $\tau = t + \Delta$ is measurable with respect to h_t.

Given $(\mathcal{M}, w, \tau)$, an *assessment* (a, m) consists of an effort strategy a for A, a report strategy m for P, and a system of beliefs. a consists of an effort choice a_t for each t depending on $h_{t-\Delta}$ and A's private history H_t^A of effort choices and private randomizing devices. m consists of a message choice m_t for each t depending on $h_{t-\Delta}$ and P's private history H_t^P of observations $\{dX_s\}_{s \le t}$ and private randomizing devices. The system of beliefs consists of a belief about $H_{t-\Delta}^P$ at each decision node $(H_t^A, h_{t-\Delta})$ of A, and a belief about H_t^{A+} which consists of H_t^A and A's date t effort choice at each decision node $(H_t^P, h_{t-\Delta})$ of P.

A *contract* $(\mathcal{M}, w, \tau, a, m)$ is a contract game plus an assessment. Given a contract, the date t continuation payoffs of A and P at the beginning of date t are

$$W_t(H_{t-\Delta}^{A+}, h_{t-\Delta}) = \mathbf{E}_t^A \left[\sum_{t \le s < \tau} e^{-r(s-t)}(w_s - h(a_s)\Delta) + e^{-r(\tau - t)} w_\tau \right],$$

$$V_t(H_{t-\Delta}^{P}, h_{t-\Delta}) = \mathbf{E}_t^P \left[\sum_{t \le s < \tau} e^{-r(s-t)}(-w_s + u(dX_s)) - e^{-r(\tau - t)} w_\tau \right].$$

$\mathbf{E}_t^A$ is with respect to the distribution over $H_{t-\Delta}^P$ generated by A's date t randomizing device and A's beliefs about $H_{t-\Delta}^P$ at all decision nodes $(H_t^A, h_{t-\Delta})$ succeeding $(H_{t-\Delta}^{A+}, h_{t-\Delta})$. $\mathbf{E}_t^P$ is with respect to the distribution over $H_{t-\Delta}^{A+}$ generated by P's belief about $H_{t-\Delta}^{A+}$ at his decision node $(H_{t-\Delta}^P, h_{t-\Delta})$.

2.2 Incentive Compatibility and Equilibrium Selection

Incentive compatibility typically means the assessment is a sequential equilibrium. However, when monitoring is private, many sequential equilibria feature implausible behavior by the principal. I refine sequential equilibria based on an idea of Dewatripont (1987) for selecting perfect equilibria in models of spatial competition:

Imagine two firms sequentially choose distinct locations on a circle representing the geography of an arbitrarily fine but discrete market of consumers. Before play begins, an incumbent firm is already located at the top of the circle. Each consumer buys from the firm located closest to him and a firm's payoff is his market share.

This game has many perfect equilibria. Given firm 1's location choice, firm 2's multiple best responses comprise the longer of the two arcs between the incumbent firm and firm 1. Each strategy firm 2 can use to select among his best responses leads to a different perfect equilibrium. Are all of these perfect equilibria plausible?

The largest market share firm 2 can capture in any perfect equilibrium is $\frac{1}{2}$. It is the outcome if, for example, firm 2 uses the following strategy, l, to select among his best responses: Choose the midpoint best-response if firm 1 locates right next to the incumbent, otherwise choose the best response location right next to firm 1.

Dewatripont (1987) argues firm 2 should be able to secure his most-preferred outcome of capturing half of the market by committing to use a strategy like l no matter what firm 1 does. Therefore, the only plausible perfect equilibria are ones in which firm 2 captures half of the market. One can think of the act of firm 2 committing to l as firm 2 making a threat before play begins that is intended to induce firm 1 into locating right next to the incumbent. Firm 1 should treat firm 2's threat as credible: After all, firm 2 is not threatening to do anything in the future that his future self would not want to do.

This idea of requiring players to take advantage of available utility-maximizing credible threats can be used to refine sequential equilibria in my model.

In general, given a contract, P can have many best response messages at the end of date t all maximizing his own date $t+\Delta$ continuation payoff while delivering different date $t+\Delta$ continuation payoffs to A. This mirrors how firm 2 has many best response locations that all maximize his own market share while delivering different market shares to firm 1. Now, just as firm 2's choice of strategy for how to select among his best responses affects his own payoff by affecting firm 1's location choice in the spatial competition model, P's choice of strategy for how to select among his date t best responses affects his own date t continuation payoff by affecting how much date t effort A exerts in my model. P's most preferred outcome standing at the beginning of date t is the one that induces the most date t effort from A. To secure this outcome, P, in general, needs to commit to a date t report strategy under which A's date $t+\Delta$ continuation payoff depends on P's private information only up to dX_t – dependence on anything else, which is unaffected by date t effort, will only dilute incentives. This is the equivalent of firm 2 committing to use a strategy like l. As I will show, this simple constraint on P's report strategy has far-reaching consequences for optimal contracting and the relationship between monitoring quality and productivity.

My strategy for developing a credible threats equilibrium concept is to be conservative when using the idea of Dewatripont (1987) to remove sequential equilibria. This way when I do remove an equilibrium, it is hard to object. The one potential downside to a conservative approach is that one might be able to come up with a reasonable stronger refinement based on the same credible threats idea, and if the two refinements lead to different predictions about optimality then who is to say my credible threats equilibrium concept is the right one? However, I show that given a contract game, the set of equilibria that survive my weak refinement all generate the same continuation payoff process. This means no matter how my weak refinement is strengthened, as long as the strengthening does not remove all equilibria from a contract game then Pareto-optimal contracts are unchanged.

Assumption 1. *Let R be any non-empty, finite set of real numbers. Let ξ be any full-support finite-valued random variable whose distribution does not depend on a_t. There exists a unique function $f^R(dX_t)$ taking values in R with the following two properties:*

- *The set* $\arg\max_{a_t} \mathbf{E}_{a_t} f^R(dX_t) - h(a_t)\Delta$ *contains a single element a^R.*
- *For any function $g(dX_t, \xi)$ taking values in R, if it is not true that $g(dX_t, \xi) = f^R(dX_t)$ for all dX_t and ξ, then a^R is strictly larger than any element of* $\arg\max_{a_t} \mathbf{E}_{a_t,\xi} g(dX_t, \xi) - h(a_t)\Delta$.

One can think of R as a set of possible rewards for A, g as a performance-sensitive reward function designed to induce effort from A, and ξ as noise. When Assumption 1 is used in the analysis below, R will correspond to the set of possible discounted date $t + \Delta$ continuation payoffs for A, g will be P's date t report strategy, and ξ will be P's private history leading up to date t. Assumption 1 says to maximize effort A's performance-sensitive reward cannot depend on noise.

Assumption 1 holds under many natural models of how effort affects the distribution of dX_t, such as if *effort has a monotone effect on* dX_t: $Im(dX_t)$ can be divided into disjoint subsets *Good* and *Bad* such that $\mathbf{P}(dX_t = x \mid a_t)$ is strictly increasing (decreasing) in a_t if and only if $x \in Good$ ($x \in Bad$). In this case, f^R takes at most two values, the maximal and minimal values of R, with f^R taking the minimal value of R if and only if $dX_t \in Bad$.

To operationalize my conservative approach to removing equilibria based on the credible threats idea, I begin by defining some restrictive conditions on assessments that will need to be satisfied for there to be a credible threats opportunity.

Definition. $W_t(H^{A+}_{t-\Delta}, h_{t-\Delta})$ *is belief-free if it does not depend on A's beliefs at all succeeding* $(H^A_t, h_{t-\Delta})$. *W_t is public given $h_{t-\Delta}$ if $W_t(H^{A+}_{t-\Delta}, h_{t-\Delta})$ is constant across all $H^{A+}_{t-\Delta}$, in which case I simplify $W_t(H^{A+}_{t-\Delta}, h_{t-\Delta})$ to $W_t(h_{t-\Delta})$. Define belief-free and public for V_t similarly.*

(a, m) *is belief-free given h_t if at every succeeding decision node the corresponding player's set of best-response continuation strategies does not depend on that player's belief.*

To define when a sequential equilibrium is removed, I suppose the set of all sequential equilibria has already been whittled down to some subset $\mathcal{E}$ using the credible threats idea. I then provide restrictive conditions as a function of $\mathcal{E}$ under which certain additional sequential equilibria can be removed.

Definition. *Fix a set sequential equilibria $\mathcal{E}$. P is said to have a credible threats opportunity at the beginning of date t given $\mathcal{E}$ and conditional on $h_{t-\Delta}$ if for every succeeding h_t, all $(a, m) \in \mathcal{E}$ are belief-free given h_t and share the same belief-free, public continuation payoff process $(W_{s+\Delta}(h_s), V_{s+\Delta}(h_s))_{s \geq t}$.*

When P has a credible threats opportunity, his set of best response messages is

$$\mathcal{M}^*(h_{t-\Delta}) := \arg\max_{m' \in \mathcal{M}} \mathbf{E}[e^{-r\Delta} V_{t+\Delta}(h_t) \mid h_{t-\Delta} m'].$$

Notice only P has the opportunity to make a credible threat, and P has a credible threats opportunity at date t only when the equilibrium property of all $(a, m) \in \mathcal{E}$ starting from date $t + \Delta$ do not depend on what happens before date $t + \Delta$ and the continuation payoff processes of A and P starting from date $t + \Delta$ are uniquely determined and do not depend on what happens before date $t + \Delta$. Thus, when P has a credible threats opportunity at date t one can think of date t as the terminal date with the players receiving lump sum payments

$$(\mathbf{E}[e^{-r\Delta} W_{t+\Delta}(h_t) \mid h_{t-\Delta} m_t], \mathbf{E}[e^{-r\Delta} V_{t+\Delta}(h_t) \mid h_{t-\Delta} m_t])$$

at the end of date t after P makes his final report m_t. By identifying these lump sum payments with market shares for firms 1 and 2, P's strategic situation at the beginning of date t becomes identical to that of firm 2 at the beginning of the spatial competition game.

Complementing my conservative approach to deciding when a player can make a credible threat is my conservative approach to defining what is a credible threat:

Definition. *Suppose P has a credible threats opportunity given $\mathcal{E}$ and conditional on $h_{t-\Delta}$. A credible threat $\hat{m}_t(h_{t-\Delta})$ is a choice of a message $\in \mathcal{M}^*(h_{t-\Delta})$ for each $(H_t^P, h_{t-\Delta})$ that depends on H_t^P only up to dX_t.*

Given a credible threat, A's best response effort does not depend on A's belief about P's private history and is, therefore, public. Consequently, P's date t continuation payoff from making the credible threat does not depend on P's belief about A's private history and is, therefore, belief-free and public:

Define $a_t | \hat{m}_t(h_{t-\Delta})$ to be the largest element of

$$\arg\max_{a'} \mathbf{E}_{a', \hat{m}_t(h_{t-\Delta})} \left[-h(a')\Delta + e^{-r\Delta} W_{t+\Delta}(h_t) \right]$$

where the expectation is computed using the distribution over the set of h_t compatible with $h_{t-\Delta}$ generated by a date t effort a' and $\hat{m}_t(h_{t-\Delta})$. Define

$$V_t(h_{t-\Delta})|\hat{m}_t(h_{t-\Delta}) := \mathbf{E}_{a_t|\hat{m}_t(h_{t-\Delta}),\hat{m}_t(h_{t-\Delta})}\left[u(dX_t) + e^{-r\Delta}V_{t+\Delta}(h_t)\right].$$

I now implicitly define when an equilibrium can be removed by defining when a set of equilibria can no longer be further refined:

Definition. *A set $\mathcal{E}$ of sequential equilibria satisfies the credible threats property if whenever P has a credible threats opportunity conditional on $h_{t-\Delta}$, there does not exist an $(a,m) \in \mathcal{E}$, $H^P_{t-\Delta}$, and a credible threat $\hat{m}_t(h_{t-\Delta})$ such that $V_t(h_{t-\Delta})|\hat{m}_t(h_{t-\Delta}) > V_t(H^P_{t-\Delta}, h_{t-\Delta})$.*

The order in which equilibria are removed under my conservative approach does not matter:

Lemma 1. *If $\mathcal{E}_1$ and $\mathcal{E}_2$ are sets of sequential equilibria that satisfy the credible threats property, then so is $\mathcal{E}_1 \cup \mathcal{E}_2$. Thus, there is a unique maximal set $\mathcal{E}^*$ of sequential equilibria that satisfies the credible threats property.*

Proof. See appendix. □

Definition. *A credible threats equilibrium is defined to be an element of $\mathcal{E}^*$.*

Proposition 1. *Fix a contract game. All $(a,m) \in \mathcal{E}^*$ are belief-free and generate the same belief-free, public continuation payoff process that can be computed recursively:*

When $\tau = t$, all $(a,m) \in \mathcal{E}^*$ generate the same belief-free, public continuation payoff $(W_t(h_{t-\Delta}), V_t(h_{t-\Delta})) = (w_t(h_{t-\Delta}), -w_t(h_{t-\Delta}))$. If $\tau > t$, then by induction suppose all $(a,m) \in \mathcal{E}^*$ generate the same belief-free, public continuation payoff $(W_{t+\Delta}(h_t), V_{t+\Delta}(h_t))$ for all h_t. Define

$$R(h_{t-\Delta}) := \{\mathbf{E}[e^{-r\Delta}W_{t+\Delta}(h_t) \mid h_{t-\Delta}m'] \mid m' \in \mathcal{M}^*(h_{t-\Delta})\}.$$

Then $m_t(H^P_t, h_{t-\Delta}) = f^{R(h_{t-\Delta})}(dX_t)$, $a_t(h_{t-\Delta}) = a^{R(h_{t-\Delta})}$, and

$$W_t(h_{t-\Delta}) = w_t(h_{t-\Delta}) - h\left(a^{R(h_{t-\Delta})}\right)\Delta + e^{-r\Delta}\mathbf{E}_{a^{R(h_{t-\Delta})},\ f^{R(h_{t-\Delta})}(dX_t)}W_{t+\Delta}(h_t),$$
$$V_t(h_{t-\Delta}) = -w_t(h_{t-\Delta}) + \mathbf{E}_{a^{R(h_{t-\Delta})},\ f^{R(h_{t-\Delta})}(dX_t)}\left[u(dX_t) + e^{-r\Delta}V_{t+\Delta}(h_t)\right].$$

Proof. See appendix. □

Despite my conservative approach to removing sequential equilibria, Proposition 1 implies that all the "complex" sequential equilibria involving P trying to keep A in the dark about his own continuation payoff are removed.

Proposition 1 says that P's report strategy at date t is characterized by the function $f^{R(h_{t-\Delta})}$. Given the definition of f^R in Assumption 1 and given that $R(h_{t-\Delta})$ is defined to be all the possible expected discounted date $t+\Delta$ continuation payoffs for A as a function of P's date t report, Proposition 1 basically says:

Remark 1. *The principal is always maximizing the incentive power of information.*

Definition. *A contract is incentive-compatible if the assessment is a credible threats equilibrium and* $W_t(h_{t-\Delta}) + V_t(h_{t-\Delta}) \geq 0$ *for all* $h_{t-\Delta}$*.*

The second part of the definition is an interim participation constraint. If it is violated both players are strictly better off terminating at the beginning of date t under some severance pay $\hat{w}_t$.

The Optimal Contracting Problem: For each point on the Pareto-frontier, find an incentive-compatible contract that achieves it.

Theorem 1. *Every payoff on the Pareto-frontier can be achieved by a contract with the following structure:*

- $\mathcal{M} = Im(dX_t)$ *and* $m_t(H_t^P, h_{t-\Delta}) = dX_t$*.*
- *For each* $t < T$ *there is a pair of constants* $w_t^{salary}, w_{t+\Delta}^{severance}$ *such that A is paid* w_t^{salary} *at date t for working and is paid a severance* $w_{t+\Delta}^{severance}$ *at date* $t + \Delta$ *if he is terminated at the beginning of date* $t + \Delta$*. Termination at date* $t + \Delta$ *occurs with some probability* $p_t^*(dX_t)$*.*

Proof. See appendix. □

The incentive structure of Pareto-optimal contracts is transparent. At each date, P truthfully reports A's performance. P's report then determines the probability that A gets terminated. Termination is inefficient as it destroys continuation surplus. Since truthful reporting must be incentive-compatible, P's continuation payoff does not depend on his report. Thus, A bears the cost of continuation surplus destruction and prefers not to be terminated. The threat of termination then motivates A to exert effort. The mapping from dX_t to how likely A is terminated is structured to maximize the effort induced from A.

Corollary 1. *If* a_t *has a monotone effect on* dX_t *Pareto-optimal contracts can be simplified so that* $\mathcal{M} = \{pass, fail\}$*,* $m_t(H_t^P, h_{t-\Delta}) = fail$ *if and only if* $dX_t \in Bad$*, and there exists a* p_t^* *such that* $p_t^*(fail) = p_t^*$ *and* $p_t^*(pass) = 0$*.*

3 Better Monitoring/Worse Outcome

Corollary 1 hints at how better monitoring might lead to a worse outcome. Notice, the principal reports $fail$ after observing any Bad signal. Now, if Bad signals occurred rarely and were all highly indicative of lower effort, then this incentive-maximizing report strategy is aligned with maximizing efficiency. On the other hand, if most Bad signals are only marginally bad – that is, the probability of occurring decreases only slightly when effort is raised – then insisting on failing the agent whenever a *Bad* signal is realized can be quite inefficient. In this section, I show that given a monitoring technology there is a systematic way to improve it so that *Bad* signals become more common and lose their statistical power, leading to lower productivity and a worse outcome for the firm.

While the better monitoring/worse outcome result is relevant regardless of the length of a date, it emerges most forcefully in the continuous-time limit when comparative statics can be stated in the starkest terms (e.g. better monitoring drives effort to zero), and I can be quite precise about what types of improvements to monitoring are counterproductive – as well as what types of improvements are productive. For example, starting with a bad news Poisson monitoring technology under which Pareto-optimal contracts induce positive effort, adding a Brownian component causes Pareto-optimal contracts to induce zero effort. On the other hand, starting with a Brownian monitoring technology under which Pareto-optimal contracts induce zero effort, adding a component that resembles a "fat-tailed" bad news Poisson process restores positive effort under Pareto-optimal contracts.

To study Pareto-optimal contracts in the continuous-time limit, I fix a family of models parameterized by $\Delta > 0$. I focus on the case when effort has a monotone effect on dX_t. To ensure that the limiting behavior of Pareto-optimal contracts is well-defined, I assume the family of models has the following limits: For all a_t,

$$\begin{aligned}\lim_{\Delta\to 0} \mathbf{E}_{a_t} u(dX_t) &= \Theta(\Delta)\\ \lim_{\Delta\to 0} \frac{d}{da_t}\mathbf{P}(dX_t \in Good \mid a_t) &= \Theta(\Delta^{\alpha}) \text{ for some } \alpha \geq 0\\ \lim_{\Delta\to 0} \mathbf{P}(dX_t \in Bad \mid a_t) &= \Theta(\Delta^{\gamma^b}) \text{ for some } \gamma^b \geq 0\\ \lim_{\Delta\to 0} \mathbf{P}(dX_t \in Good \mid a_t > 0) &= \Theta(\Delta^{\gamma^g}) \text{ for some } \gamma^g \geq 0.\end{aligned}$$

Obviously, $\alpha \geq \max\{\gamma^b, \gamma^g\}$, and γ^g or γ^b is equal to zero. α measures the incentive power of information – the lower is α the more effort that can be induced. $\alpha - \gamma^b$ is the exponent associated with the elasticity of the probability of *Bad* signals with respect to effort. The smaller is $\alpha - \gamma^b$, the more informative a typical *Bad* signal is of lower effort. However, it would be incorrect to think of $\alpha - \gamma^b$ as measuring the statistical power of information. Even if $\alpha - \gamma^b$ is quite large, there can still be individual *Bad* signals that are quite informative of lower effort.

The class of models being considered encompasses a broad range of familiar settings, including the cases where effort affects the drift of a Brownian motion:

$$dX_t = \begin{cases} \sqrt{\Delta} & \text{with probability } \frac{1}{2} + \frac{a_t\sqrt{\Delta}}{2} \\ -\sqrt{\Delta} & \text{with probability } \frac{1}{2} - \frac{a_t\sqrt{\Delta}}{2} \end{cases}$$

the intensity of a good news Poisson process:

$$dX_t = \begin{cases} g & \text{with probability} \quad a_t \Delta\lambda \\ b & \text{with probability} \quad 1 - a_t \Delta\lambda \end{cases}$$

and the intensity of a bad news Poisson process:

$$dX_t = \begin{cases} g & \text{with probability} \quad 1 - (1-a_t)\Delta\lambda \\ b & \text{with probability} \quad (1-a_t)\Delta\lambda \end{cases}$$

Definition. *dX_t is Brownian if $(\alpha, \gamma^b, \gamma^g) = (.5, 0, 0)$, good news Poisson if $(\alpha, \gamma^b, \gamma^g) = (1, 0, 1)$, bad news Poisson if $(\alpha, \gamma^b, \gamma^g) = (1, 1, 0)$, and fat-tailed bad news Poisson if $(\alpha, \gamma^b, \gamma^g) = (\alpha, \alpha, 0)$ for $\alpha \in (0, 1)$.*

Theorem 2. *If $\alpha - \gamma^b > 0$ then Pareto-optimal contracts induce zero effort for all sufficiently small Δ. If $\alpha - \gamma^b = 0$ and $\alpha \leq 1$ then the model can be parameterized so that Pareto-optimal contracts continue to induce non-zero effort as $\Delta \to 0$.*

Proof. See appendix. □

Given that $\alpha - \gamma^b$ measures just how bad is a typical *Bad* signal (lower value means more bad), Theorem 2 formalizes an intuition sketched out at the beginning of this section: When a typical *Bad* signal is only marginally bad, Pareto-optimal contracts are far from efficient, inducing zero effort.

Corollary 2. *If dX_t is Brownian or good news Poisson, Pareto-optimal contracts induce zero effort as $\Delta \to 0$. If dX_t is bad news Poisson, there are parameterizations of the model under which Pareto-optimal contracts induce nonzero effort as $\Delta \to 0$.*

This corollary matches classic results from the literature on repeated games with public monitoring. For example, Abreu, Milgrom, and Pearce (1991) show that in a continuous-time repeated prisoner's dilemma game with public monitoring cooperation can be supported as an equilibrium if monitoring is bad news Poisson but not good news Poisson. Sannikov and Skrzypacz (2007) show that in a continuous-time repeated Cournot oligopoly game with public monitoring collusion cannot be supported if monitoring is Brownian. This common baseline allows me to highlight how the relationship between intra-firm monitoring and productivity/firm value in my model is different than the relationship between inter-firm monitoring and collusion/firm value in the repeated games literature, where improvements to the information content of monitoring always weakly improve the scope for collusion and the value of the firm.

Armed with Theorem 2 I can investigate how improvements to the monitoring technology affect optimality. I begin with a binary valued monitoring technology $dX_{1t} \in \{b_1, g_1\}$ with associated exponents $(\alpha_1, \gamma_1^b, \gamma_1^g)$. I then improve it by adding a conditionally independent binary valued signal $dX_{2t} \in \{b_2, g_2\}$ with associated exponents $(\alpha_2, \gamma_2^b, \gamma_2^g)$. I show that it is generically the case that effort has a monotone effect on the vector valued information (dX_{1t}, dX_{2t}) generated by the improved monitoring technology. Thus, (dX_{1t}, dX_{2t}) also has some associated exponents $(\alpha, \gamma^b, \gamma^g)$. I derive the formulas for α, γ^b, and γ^g as a function of $(\alpha_1, \gamma_1^b, \gamma_1^g)$ and $(\alpha_2, \gamma_2^b, \gamma_2^g)$. Then, by inverting the formulas and using Theorem 2, I can show, given $(\alpha_1, \gamma_1^b, \gamma_1^g)$, what kinds of improvements $(\alpha_2, \gamma_2^b, \gamma_2^g)$ are counterproductive and what kinds are productive.

The vector valued (dX_{1t}, dX_{2t}) can take one of four values: $(g_1, g_2), (g_1, b_2), (b_1, g_2)$ and (b_1, b_2). Holding Δ fixed, $\mathbf{P}((dX_{1t}, dX_{2t}) = (g_1, g_2) \mid a_t, \Delta)$ is strictly increasing in a_t and $\mathbf{P}((dX_{1t}, dX_{2t}) = (b_1, b_2) \mid a_t, \Delta)$ is strictly decreasing in a_t. The probability that $(dX_{1t}, dX_{2t}) = (g_1, b_2)$ is $\mathbf{P}(dX_{1t} = g_1 \mid a_t, \Delta) \cdot \mathbf{P}(dX_{2t} = b_2 \mid a_t, \Delta)$. By the product rule, as $\Delta \to 0$, the derivative of $\mathbf{P}((dX_{1t}, dX_{2t}) = (g_1, b_2) \mid a_t, \Delta)$ with respect to a_t is $A(\Delta) - B(\Delta)$ where $A(\Delta) = \Theta(\Delta^{\alpha_1 + \gamma_2^b})$ and $B(\Delta) = \Theta(\Delta^{\gamma_1^g + \alpha_2})$. A sufficient condition for $\mathbf{P}((dX_{1t}, dX_{2t}) = (g_1, b_2) \mid a_t, \Delta)$ to be a monotonic function of a_t in the continuous-time limit is $\alpha_1 + \gamma_2^b \neq \gamma_1^g + \alpha_2$. Similarly, a sufficient condition for $\mathbf{P}((dX_{1t}, dX_{2t}) = (b_1, g_2) \mid a_t, \Delta)$ to be a monotonic function of a_t in the continuous-time limit is $\alpha_1 + \gamma_2^g \neq \gamma_1^b + \alpha_2$. Thus,

Lemma 2. *If $\alpha_1 - \alpha_2 \neq \gamma_1^g - \gamma_2^b$ or $\gamma_1^b - \gamma_2^g$ then effort has a monotone effect on (dX_{1t}, dX_{2t}) as $\Delta \to 0$.*

Proposition 2. *Given dX_{1t} and dX_{2t} with associated exponents $(\alpha_1, \gamma_1^b, \gamma_1^g)$ and $(\alpha_2, \gamma_2^b, \gamma_2^g)$, if $\alpha_1 \geq \alpha_2$ then the associated exponents of the vector-valued (dX_{1t}, dX_{2t}) are*

$$\begin{aligned}
(\alpha = \alpha_2, \gamma^b = \min\{\gamma_1^b, \gamma_2^b\}, \gamma^g = \gamma_2^g) \quad &\text{if } \gamma_1^g - \gamma_2^b < \alpha_1 - \alpha_2 < \gamma_1^b - \gamma_2^g \\
(\alpha = \alpha_2, \gamma^b = \gamma_2^b, \gamma^g = \min\{\gamma_1^g, \gamma_2^g\}) \quad &\text{if } \gamma_1^b - \gamma_2^g < \alpha_1 - \alpha_2 < \gamma_1^g - \gamma_2^b \\
(\alpha = \alpha_2, \gamma^b = \gamma_2^b, \gamma^g = \gamma_2^g) \quad &\text{if } \gamma_1^g - \gamma_2^b,\ \gamma_1^b - \gamma_2^g < \alpha_1 - \alpha_2
\end{aligned}$$

Proposition 2 only considers the case where $\alpha_1 \geq \alpha_2$. The other case, $\alpha_2 \geq \alpha_1$, is implied by symmetry.

Proof. See appendix. □

Proposition 2 yields explicit characterizations of counterproductive and productive improvements to the monitoring technology.

Corollary 3. *Suppose $\alpha_1 - \gamma_1^b = 0$. If $\alpha_2 - \gamma_2^b > 0$, $\gamma_2^b < \gamma_1^b$, and $\alpha_2 < \alpha_1 + \gamma_2^b$ then $\alpha - \gamma^b > 0$.*

Corollary 3 implies that improving bad news Poisson monitoring by adding a Brownian component causes Pareto-optimal contracts to induce zero effort.

Given a monitoring technology dX_{1t} under which Pareto-optimal contracts induce positive effort, Corollary 3 lists three intuitive conditions that, if satisfied by the new technology dX_{2t} being added, leads to the worst outcome in which Pareto-optimal contracts induce zero effort. The first condition is that the new technology itself has weak *Bad* signals: $\alpha_2 - \gamma_2^g > 0$. Intuitively, improving a monitoring technology that generates strong *Bad* signals with more strong *Bad* signals should not lead to weak *Bad* signals. Indeed, Proposition 2 implies that if both $\alpha_1 - \gamma_1^b = 0$ and $\alpha_2 - \gamma_2^b = 0$ then $\alpha - \gamma^b = 0$. Even if dX_{2t} does have weak *Bad* signals they have to be sufficiently common for there to be a tangible effect on the *Bad* signals of the combined monitoring technology (dX_{1t}, dX_{2t}). This is the second condition. Indeed, Proposition 2 implies that if $\gamma_2^b \geq \gamma_1^b$, then no matter how weak are the *Bad* signals of dX_{2t}, the *Bad* signals of (dX_{1t}, dX_{2t}) have approximately the same informativeness as that of dX_{1t} which are assumed to be strong: If $\gamma_2^b > \gamma_1^b$, then $\gamma = \gamma_1^b$, $\alpha = \alpha_1$, and therefore $\alpha - \gamma = 0$. Finally, even if the *Bad* signals of dX_{2t} are weak and common enough to be able to tangibly affect the informativeness of the *Bad* signals of (dX_{1t}, dX_{2t}), if the principal ignores dX_{2t} then nothing changes. By Remark 1, we know the principal's report strategy is dictated by the incentive power of information. Thus, the third condition ensures that the incentive power of dX_{2t} is sufficiently strong so that the principal will not ignore dX_{2t}. Indeed, the proof of Proposition 2 shows that as α_2 crosses the threshold $\alpha_1 + \gamma_2^b$ from below the principal's incentive-maximizing report strategy shifts from reporting $fail$ whenever $dX_{1t} = b_1$ or $dX_{2t} = b_2$ to ignoring dX_{2t} and reporting $fail$ only when $dX_{1t} = b$.

It is worth emphasizing that the upper bound on α_2 is $\alpha_1 + \gamma_2^b$, not α_1: The new technology has to have sufficiently strong incentive power but it does not necessarily have to have stronger incentive power than the original monitoring technology. Proposition 2 implies that $\alpha = \min\{\alpha_1, \alpha_2\}$. While this result reinforces Remark 1 about how the principal is always maximizing incentive power, it gives the incorrect impression that the principal just picks the dX_{it} with the highest incentive power and reports off of that, ignoring the other dX_{-it}. If that were the case then improving monitoring with adding a dX_{2t} with $\alpha_2 > \alpha_1$ could never cause Pareto-optimal contracts to collapse, contradicting Corollary 3.

Corollary 3 and the discussion that followed suggests the following lesson about the dangers of improving monitoring:

Remark 2. *Improving a functional monitoring system by introducing technology with weak bad signals is neutral at best and can be extremely hurtful.*

Corollary 4. *Suppose* $\alpha_1 > \gamma_1^b$. *If* $\alpha_2 - \gamma_2^b = 0$ *and* $\alpha_2, \gamma_2^b < \max\{\alpha_1 - \gamma_1^b, \gamma_1^b\}$ *then* $\alpha - \gamma^b = 0$.

Given a monitoring technology dX_{1t} under which Pareto-optimal contracts induce zero effort, Corollary 3 lists three intuitive conditions that, if satisfied by the new technology dX_{2t} being added, leads to the better outcome in which the monitoring technology is such that Pareto-optimal contracts can induce positive effort. The first condition is that the new technology itself has strong *Bad* signals: $\alpha_2 - \gamma_2^g = 0$. Intuitively, improving a monitoring technology that generates weak *Bad* signals with more weak *Bad* signals should not lead to strong *Bad* signals. Indeed, Proposition 2 implies that if both $\alpha_1 - \gamma_1^b > 0$ and $\alpha_2 - \gamma_2^b > 0$ then $\alpha - \gamma^b > 0$. In particular, if the current monitoring technology does not support positive effort, investing in acquiring additional Brownian or good news Poisson information does not help. Now just like before, even if the new monitoring technology has the right type of *Bad* signals (in this case, strong *Bad* signals), in order to have an impact, the principal must be willing to pay attention to it which requires sufficient incentive power and the signals must be sufficiently common. This leads to the second and third conditions, $\alpha_2, \gamma_2^b < \max\{\alpha_1 - \gamma_1^b, \gamma_1^b\}$.

Together, Corollary 4 and Theorem 2 imply that improvements that are potentially productive all have exponents of the form $(\alpha_2, \alpha_2, 0)$ where $\alpha_2 \in [0, 1]$. Moreover, if adding a monitoring technology with exponents $(x, x, 0)$ is productive, then so is adding any monitoring technology with exponents $(y, y, 0)$ where $y < x$. Lastly, when $\alpha_1 > \gamma_1^b$, it cannot be that both $\alpha_1 - \gamma_1^b$ and γ_1^b are zero. Thus, if $\alpha_1 > \gamma_1^b$ there is always an improvement involving a fat-tailed Poisson monitoring technology that restores positive effort and improves firm outcome.

Remark 3. *Improvements to monitoring should generate strong bad signals that are not too rare.*

For example, suppose initially the firm has a Brownian monitoring technology that causes Pareto-optimal contracts to induce zero effort. By Corollary 4 a productive improvement is to introduce a fat-tailed Poisson component with incentive power $\alpha_2 < 0.5$. Such a technology generates informative *Bad* signals that are much rarer than those of the previous Brownian technology but still much more common that the extremely rare *Bad* signals of a bad news Poisson technology that would be of no value in the current setting. The proof of Theorem 2 then implies that the probability of termination conditional on a $fail$ report is $= \Theta(\Delta^{1-\alpha_2})$. Since $\alpha_2 < 0.5$, $1-\alpha_2 > 0$. This means, as Δ becomes small, the probability of termination even conditional on a $fail$ report becomes small.

3.1 Delay

While Corollary 4 explores what types of improvements to the information content of intra-firm monitoring can improve productivity/firm value, one of the key insights coming from the repeated games literature on inter-firm monitoring is that releasing information in batches can also improve collusion/firm value. Essentially, a batch of signals is like a single signal but with greater statistical power. Since statistical power is what matters in those models of inter-firm monitoring, the benefits of batching signals is implied. Does the same principle hold in my model of intra-firm monitoring where the incentive power of information is what matters? Even though my model, as it is currently defined, does not allow the contract to control when the principal sees dX_t, my analysis has already indirectly provided an answer to the question.

Releasing information in batches is equivalent to restricting the players to respond to new information only every once in a while. Unlike the repeated games literature where the game is taken for granted, the principal and agent in my model are doing optimal contracting and can choose the structure of the contract game. In particular, they can choose to use a contract game that only allows the principal to react to new information every once in a while: For example, the contract game could be structured so that pay and termination do not depend on any report made between t_1 and t_2. In this case, the contract game does not allow the principal to react to new information between t_1 and t_2. Since Theorem 2 is a result about Pareto-optimal contracts, contract games that react to new information only every once in a while are already folded into the analysis. Thus, my optimality result indirectly implies that delaying the release of information cannot help.

In fact, choosing a contract game that allows the principal to react to new information only every once in a while is not only not helpful, it is usually hurtful. Suppose the contract game does not allow the principal to react to new information between t_1 and t_2. On date $t_2 + \Delta$ when P finally has the opportunity to affect A's continuation payoff through his reports, all of A's efforts on or before date t_2 have been sunk. Thus, by the credible threats refinement, P will report in a way so that A's date $t_2 + 2\Delta$ continuation payoff depends only on $dX_{t_2+\Delta}$. Anticipating this, A exerts zero effort from t_1 to t_2. More generally, on any date t where P's report has no payoff impact, A's effort is zero.

The analysis in this section shows how the relationship between monitoring quality and firm value in my model is fundamentally different than the relationship between monitoring quality and firm value in the repeated games literature. In the repeated games literature, what matters is the statistical power of information. Because of this, improving the information content of monitoring holding fixed monitoring frequency never hurts firm value and delaying the frequency of monitoring holding fixed the information content of monitoring often increases firm value. In my model, what matters is the incentive power of information. This leads to comparative statics that are almost the opposite of those in the repeated games literature: Improving the information content of monitoring holding fixed monitoring frequency often hurts firm value and delaying the frequency monitoring holding fixed the information content of monitoring never increases firm value.

4 Monitoring Design

The previous section's analysis shows that the way monitoring is conducted has a significant impact on worker productivity and, ultimately, firm value. This leads naturally to the subject of monitoring design.

If the principal has complete freedom in choosing a monitoring technology ex-ante, a folk theorem can be proved. Let S^* denote the first-best flow surplus and S^*_T denote the flow surplus of Pareto-optimal contracts as a function of T, holding Δ fixed.

Proposition 3. *For all r sufficiently close to zero,* $\lim_{T\to\infty} S^*_T = S^*$.

In practice monitoring is costly and there may be technological constraints that effectively make some monitoring technologies infinitely costly. A thorough understanding of the technological frontier of monitoring is beyond the scope of this paper. The better monitoring/worse outcome result of the previous section implies that even in a world with costly monitoring, the optimal structure of monitoring need not be determined by cost at the margin.

In a setting where monitoring involves sampling an exogenously fixed stochastic process tracking the cumulative productivity of the agent, the frequency of monitoring determines its information content. Optimal monitoring design then becomes a problem of designing the optimal sequence of random times for the principal to monitor. I now consider a canonical example of a model with this type of monitoring: A exerts effort affecting the drift of a Brownian motion and monitoring by P is privately observing the current value of the Brownian motion. The corollary to Theorem 2 implies Pareto-optimal contracts do not induce positive effort if monitoring occurs continuously. In this section, I expand the optimal contracting problem by allowing contracts to specify when the principal can randomly monitor. I show that if the sequence of monitoring times is required to be predictable then optimal monitoring occurs on evenly spaced, non-random dates, resulting in Pareto-optimal contracts that generate positive but cyclic worker productivity.

In the current model, the timing of events at each date t is the same as before except P may or may not monitor A. If P does monitor A, the private informative signal he observes is no longer dX_t where

$$dX_t = \begin{cases} \mu\Delta + \sqrt{\Delta} & \text{with probability} \quad \frac{1}{2} + \frac{a_t\sqrt{\Delta}}{2} \\ \mu\Delta - \sqrt{\Delta} & \text{with probability} \quad \frac{1}{2} - \frac{a_t\sqrt{\Delta}}{2} \end{cases}$$

but, rather, $X_t = \sum_{s\leq t} dX_s$. Assume P always monitors on date $T - \Delta$.

In addition to specifying $\mathcal{M}$, w, and τ, a contract game also specifies a monitoring design e, consisting of a predictable sequence of random monitoring times $e_1 < e_2 < \ldots < T - \Delta$. Predictable means that e_{i+1} is measurable with respect to h_{e_i}. An assessment is defined similarly to before except P's decision nodes only occur on monitoring dates. Nevertheless, for each date t, W_t and V_t can be defined similar to before. In this section, I am interested in the limit as $\Delta \to 0$, and all of the analysis will assume an infinitesimal Δ.

Because the current model is a generalization of the original model, the original credible threats equilibrium concept also needs to be generalized. The same conservative approach as before can be used to develop the notion of credible threats equilibrium in the current model. In the special case when the contract game has the principal monitor each date, the current model's credible threats equilibria will reduce to the original model's credible threats equilibria.

I begin by proving that the current model's monitoring technology satisfies something akin to Assumption 1.

Lemma 3. *Let R be any non-empty, finite set of real numbers. Let ξ be any full-support finite valued random variable whose distribution does not depend on the sequence of efforts $a_{(s,t]}$ from date $s+\Delta$ through date t. There exists a unique function $f^{R,\ t-s}(X_t - X_s)$ taking values in R with the following two properties:*

- *The set* $\arg\max_{a_{(s,t]}} e^{-r(t-s)}\mathbf{E}_{a_{(s,t]}} f^{R,\ t-s}(X_t - X_s) - \sum_{t'=s+\Delta}^{t} e^{-r(t'-s)} h(a_{(s,t]}(t'))\Delta$ *contains a maximal element/effort sequence* $a^{R,\ t-s}$.

- *For any function $g(X_t - X_s, \xi)$ taking values in R, if it is not true that $g(X_t - X_s, \xi) = f^{R,\ t-s}(X_t - X_s)$ for all $X_t - X_s$ and ξ, then $a^{R,\ t-s}$ is strictly larger than any element of* $\arg\max_{a_{(s,t]}} e^{-r(t-s)}\mathbf{E}_{a_{(s,t]},\xi} g(X_t - X_s, \xi) - \sum_{t'=s+\Delta}^{t} e^{-r(t'-s)} h(a_{(s,t]}(t'))\Delta$.

$f^{R,\ t-s}(X_t - X_s)$ is characterized by a threshold $\rho^{R,\ t-s}$ such that

$$f^{R,\ t-s}(X_t - X_s) = \begin{cases} \max_{w\in R} w & \text{if } X_t - X_s > \rho^{R,\ t-s} \\ \min_{w\in R} w & \text{if } X_t - X_s \leq \rho^{R,\ t-s} \end{cases}$$

Moreover, $\arg\max_{a_{(s,t]}} e^{-r(t-s)}\mathbf{E}_{a_{(s,t]}} f^{R,\ t-s}(X_t - X_s) - \sum_{t'=s+\Delta}^{t} e^{-r(t'-s)} h(a_{(s,t]}(t'))\Delta$ *generically has a single element. When the set does not have a single element, decreasing* $\min_{w\in R} w$ *infinitesimally will cause the set to have a single element that is infinitesimally close to* $a^{R,\ t-s}$.

Proof. See appendix. □

Lemma 3 speaks of higher and lower effort sequences. This is well-defined since first-order conditions imply that given two best response effort sequences $a_{(s,t]}$, $a'_{(s,t]}$, if $a_{(s,t]}(t') > a'_{(s,t]}(t')$ for some $t' \in (s,t]$ then $a_{(s,t]}(t') > a'_{(s,t]}(t')$ for all $t' \in (s,t]$.

I will use Lemma 3 to show that at the end of a monitoring period running from $s+\Delta$ to t the principal's report strategy is characterized by a function $f^{R,\ t-s}$. This mirrors what I did in the original model except I did not have to worry about the agent having multiple best response efforts. I will use the second half of Lemma 3 to justify assuming that if the agent has multiple best response effort sequences, he will choose the maximal effort sequence $a^{R,\ t-s}$ which is most preferred by the principal. This is because the second half of Lemma 3 implies that the principal can ensure something close to $a^{R,\ t-s}$ is the unique best response of the agent by reducing the agent's outside option by ε.

The definitions of belief-free continuation payoff, public continuation payoff, and belief-free (a, m) are all unchanged.

Definition. *Fix a set of sequential equilibria $\mathcal{E}$ and a public history h_s such that P monitored at date s given h_s and t is the next monitoring date. P is said to have a credible threats opportunity at the beginning of date $s+\Delta$ given $\mathcal{E}$ and conditional on h_s if, for every h_t compatible with h_s, all $(a,m) \in \mathcal{E}$ are belief-free given h_t and share the same belief-free, public continuation payoff $(W_{t+\Delta}(h_t), V_{t+\Delta}(h_t))$ on date $t+\Delta$ and all subsequent dates that start a monitoring period.*

When P has a credible threats opportunity, his set of best response messages is

$$\mathcal{M}^*(h_s) := \arg\max_{m' \in \mathcal{M}} \mathbf{E}[e^{-r\Delta} V_{t+\Delta}(h_t) \mid h_s m'].$$

This definition of a credible threats opportunity generalizes the original definition to the current model. Just like before, I only allow P to make credible threats. As part of my conservative approach, I only allow P to have a credible threats opportunity at the beginning of a monitoring period, which is defined to be the time period between consecutive monitoring dates, and only if at the end of the period all elements of $\mathcal{E}$ are basically the same and don't depend on what happens before conditional on the public history. In the special case when the monitoring period is a single date, the current definition of a credible threats opportunity reduces to the previous definition. The definition of $\mathcal{M}^*$ is unchanged except it is written as $\mathcal{M}^*(h_s)$ instead of $\mathcal{M}^*(h_{t-\Delta})$ since nothing public occurs during the time interval (s,t).

The definition of a credible threat $\hat{m}_t(h_s)$ is the natural generalization of the original definition of a credible threat – the choice of message at date t depends on H_t^P only up to $X_t - X_s$ instead of dX_t like before. Again, in the special case when the monitoring period is a single date this definition of a credible threat reduces to the original definition. Define $a_{(s,t]}|\hat{m}_t(h_s)$ as the largest best-response effort sequence from date $s+\Delta$ through date t given $\hat{m}_t(h_s)$. It is the analogue of $a_t|\hat{m}_t(h_{t-\Delta})$ from before. Define

$$V_{s+\Delta}(h_s)|\hat{m}_t(h_s) = \mathbf{E}_{a_{(s,t]}|\hat{m}_t(h_s), \hat{m}_t(h_s)} \left[\sum_{s<t'\leq t} e^{-r(t'-s)}(dX_{t'} - w_{t'}(h_s)) + e^{-r(t-s)} V_{t+\Delta}(h_t) \right].$$

$V_{s+\Delta}(h_s)|\hat{m}_t(h_s)$ is the analogue of $V_t(h_{t-\Delta})|\hat{m}_t(h_{t-\Delta})$ from before.

The condition for when a set $\mathcal{E}$ of sequential equilibria satisfies the credible threats property is basically the same as before. Lemma 1 continues to hold. The definition of credible threats equilibrium is unchanged.

Proposition 4. *Fix a contract game. All $(a,m) \in \mathcal{E}^*$ are belief-free and generate the same public continuation payoff process that is belief-free on each date that begins a monitoring period. The continuation payoff process can be computed recursively:*

$(W_t(h_{t-\Delta}), V_t(h_{t-\Delta})) = (w_t(h_{t-\Delta}), -w_t(h_{t-\Delta}))$ *if the contract game terminates given $h_{t-\Delta}$. Fix a date $s+\Delta$ that begins a monitoring period ending on date t. Given*

$(W_{t+\Delta}(h_t), V_{t+\Delta}(h_t))$ *for all* h_t *succeeding* h_s*, define*

$$R(h_s) := \{\mathbf{E}[e^{-r\Delta}W_{t+\Delta}(h_t) \mid h_s m'] \mid m' \in \mathcal{M}^*(h_s)\}.$$

Then $m_t(H_t^P, h_s) = f^{R(h_s),\ t-s}(X_t - X_s)$, $a_{(s,t]}(h_s) = a^{R(h_s),\ t-s}$, *and* $(W_{s+\Delta}(h_s), V_{s+\Delta}(h_s)) =$

$$\Big(\sum_{s<t'\leq t} e^{-r(t'-s)} \left[w_{t'}(h_s) - h\left(a^{R(h_s),\ t-s}(t'-s)\right)\Delta\right] + e^{-r(t-s)}\mathbf{E}_{f^{R(h_s),\ t-s}(X_t-X_s)}W_{t+\Delta}(h_t),$$
$$\sum_{s<t'\leq t} e^{-r(t'-s)} \left[-w_{t'}(h_s) + \mathbf{E}_{a^{R(h_s),\ t-s}}dX_t\right] + e^{-r(t-s)}\mathbf{E}_{f^{R(h_s),\ t-s}(X_t-X_s)}V_{t+\Delta}(h_t)\Big).$$

The proof mirrors that of Proposition 1. Proposition 4 generalizes Proposition 1 and, in particular, implies that any stronger refinement that doesn't remove all equilibria from a contract game cannot lower the Pareto-frontier.

Theorem 3. *As* $T \to \infty$*, there exist* Δ^**,* ρ^**, and* p^* *such that every point on the Pareto-frontier can be achieved by a contract with the following structure:*

- *P monitors A every Δ^* units of time.*
- $\mathcal{M} = Im(X_{\Delta^*})$ *and* $m_{k\Delta^*}(H^P_{k\Delta^*}, h_{k\Delta^*-\Delta^*}) = X_{k\Delta^*} - X_{k\Delta^*-\Delta^*}$.
- *There exist a pair of constants* $(w^{severance}, w^{salary})$ *such that while A is not yet terminated, he is paid a stream* $w^{salary}dt$*. When A is terminated, he is paid a lump sum* $w^{severance}$*. At date* $k\Delta^*$*, A is terminated with probability* p^* *if* $X_{k\Delta^*} - X_{k\Delta^*-\Delta^*} \leq \rho^*$*. Otherwise A is not terminated.*

Most of the work in proving Theorem 3 has already been done. Given that Proposition 4 implies the credible threats equilibrium concept of the original model admits a natural generalization to the current model, the second and third parts of the theorem follow naturally from Theorem 1. The only thing worth remarking on is the first part which says that in Pareto-optimal contracts, monitoring occurs on non-random dates that are evenly spaced. Suppose after some monitoring date e_i, the next monitoring date e_{i+1} is random. For each realization of e_{i+1}, there is associated to it a continuation surplus starting from date $e_i + \Delta$. Pick the realization of e_{i+1} that generates the largest continuation surplus and change the Pareto-optimal contract so that after e_i, the next monitoring date is always that realization of e_{i+1}. This increases the expected continuation surplus at date $e_i + \Delta$ which can only help increase ex-ante surplus. The evenly spaced aspect of optimal monitoring is a consequence of the infinite time horizon.

The magnitude Δ^* of the endogenously determined monitoring period length is pinned down by an intuitive tradeoff. The set of signal realizations $X_{k\Delta^*} - X_{k\Delta^*-\Delta^*} \leq \rho^*$ is similar in spirit to the *Bad* set of signal realizations when effort has a monotone effect on signals. As Δ^* increases away from 0, this set of "*Bad*" signals becomes more informative of lower effort which makes maximizing incentive power less inefficient. On the other hand, as Δ^* becomes large, discounting begins eroding the incentive power of information: In the beginning of a monitoring period, the impact of destroying continuation value in the distant future when the monitoring period concludes has little effect on the continuation payoff of the agent today. The optimal Δ^* balances the need to decrease the inefficiency of maximizing incentive power with the opposing desire to increase the incentive power of information.

If one redefines $\Delta := \Delta^*$ and $dX_t := X_t - X_{t-\Delta^*}$, then Theorem 3 becomes virtually identical to Theorem 1. Thus, one way to think about Theorem 3 is that it answers the following question: In a discrete time model, how long should a date be? One way the Pareto-optimal contracts of Theorem 3 are not identical to the discrete-time Pareto-optimal contracts of Theorem 1 is with respect to the effort profile. In the original discrete time model, there is only one effort choice per date/monitoring period. In the current model, the agent chooses effort repeatedly within a monitoring period.

Corollary 5. *The effort profile in Pareto-optimal contracts is cyclic.*

This is an immediate consequence of discounting and the fact that all efforts within a monitoring period affect the observed information at the end of the monitoring period symmetrically. More specifically, the first order condition for optimality implies effort within a monitoring period increases in the unique way such that the marginal cost of effort grows at the inverse of the agent's discount rate.

4.1 Time Consistency

In generalizing the notion of credible threats equilibrium to the current model I have so far glossed over an important issue. When a monitoring period lasts more than one date, a credible threat should be time consistent. Suppose P makes a credible threat at the beginning of the monitoring period and A takes it seriously and begins enacting his best response effort sequence. The fear is that at some point in the middle of the period, P is better off changing his strategy for how to select among his best response reports at the end of the period. If this were the case, P would not be able to commit not to change his credible threat which means the credible threat isn't really credible to begin with. Allowing P to make non time consistent credible threats and using such threats to remove sequential equilibria would not be consistent with my conservative approach to removing equilibria. This of course was not an issue previously when a monitoring period was a single date.

Below I develop a stringent condition for a credible threat to be considered time-consistent (being as stringent as possible is logically consistent with my conservative approach to removing equilibria) and show that even if P is only allowed to make time-consistent credible threats, the same set of equilibria survives. The key is to show that the report strategy characterized by $f^{R,\, t-s}$ is time-consistent.

To define the notion of a time consistent credible threat $\hat{m}_t(h_s)$, begin by defining, for any $t' \in (s,t)$, $V_{t'+\Delta}(h_s)|\hat{m}_t(h_s)$. It is the natural extension of $V_{s+\Delta}(h_s)|\hat{m}_t(h_s)$ to date $t'+\Delta$. For any $t' \in (s,t)$, define $a_{(s,t']}|\hat{m}_t(h_s)$ to be the portion of $a_{(s,t]}|\hat{m}_t(h_s)$ ranging from date $s+\Delta$ through date t'. Let $\hat{m}'_t(h_s)$ be another credible threat. Define $a_{(t',t]}|\{\hat{m}'_t(h_s), a_{(s,t']}|\hat{m}_t(h_s)\}$ to be the largest best-response effort sequence from date $t'+\Delta$ through date t given $\hat{m}'_t(h_s)$ and given that A previously chose $a_{(s,t']}|\hat{m}_t(h_s)$ in the current monitoring period. Define $V_{t'+\Delta}(h_s)|\{\hat{m}'_t(h_s), a_{(s,t']}|\hat{m}_t(h_s)\}$ to be P's date $t'+\Delta$ continuation payoff under credible threat $\hat{m}'_t(h_s)$ assuming A previously chose $a_{(s,t']}|\hat{m}_t(h_s)$ in the current monitoring period.

Definition. *A credible threat $\hat{m}_t(h_s)$ is time-consistent if there does not exist a date $t' \in (s,t)$ and credible threat $\hat{m}'_t(h_s)$ such that $V_{t'+\Delta}(h_s)|\{\hat{m}'_t(h_s), a_{(s,t']}|\hat{m}_t(h_s)\} > V_{t'+\Delta}(h_s)|\hat{m}_t(h_s)$*

One can now define time-consistent credible threats equilibrium by using the definition of credible threats equilibrium except replace credible threats with time-consistent credible threats.

Lemma 4. *Given a contract game the time-consistent credible threats equilibria coincide with the credible threats equilibria.*

5 Conclusion

This paper studied how changes to the information content of private monitoring inside a firm impacts worker productivity and firm value. I showed that making monitoring better by introducing information that is strong in incentive power but weak in statistical power can backfire, leading to a decline in productivity and firm value. In some cases, improvements to monitoring can cause Pareto-optimal contracts to collapse into trivial contracts that induce zero effort. Delaying the firm's ability to react to the information generated by monitoring only makes things worse. On the other hand, improving monitoring by introducing a sufficiently fat-tailed Poisson process can be helpful.

In a setting where monitoring is sampling the current value of a fixed stochastic process tracking cumulative productivity, the frequency of monitoring determines its information content. My better monitoring/worse outcome result implies that there is value in limiting the frequency of monitoring. Optimal monitoring occurs on evenly spaced, nonrandom dates, causing worker productivity to be positive but cyclic.

6 Appendix

Proof of Proposition 1. Begin with the set $\mathcal{E}_T$ of all sequential equilibria. It is easy to verify that P has a credible threats opportunity given $\mathcal{E}_T$ and conditional on any public history of the form $h_{T-2\Delta}$ satisfying $\tau(h_{T-2\Delta}) > T - \Delta$. Now, define a new set $\mathcal{E}_{T-\Delta} \subset \mathcal{E}_T$ of sequential equilibria as follows: $(a, m) \in \mathcal{E}_{T-\Delta}$ if and only if for each $h_{T-2\Delta}$ satisfying $\tau(h_{T-2\Delta}) > T - \Delta$ there is a $f^{R(h_{T-2\Delta})}(dX_{T-\Delta})$ such that $m_{T-\Delta}(H^P_{T-\Delta}, h_{T-2\Delta}) = f^{R(h_{T-2\Delta})}(dX_{T-\Delta})$ for all $H^P_{T-\Delta}$ and $a_{T-\Delta}(H^A_{T-\Delta}, h_{T-2\Delta}) = a^{R(h_{T-2\Delta}))}$ for all $H^A_{T-\Delta}$. By construction, $\mathcal{E}^* \subset \mathcal{E}_{T-\Delta}$.

Now it is easy to verify that P has a credible threats opportunity given $\mathcal{E}_{T-\Delta}$ and conditional on any public history of the form $h_{T-3\Delta}$ satisfying $\tau(h_{T-3\Delta}) > T - 2\Delta$. Similar to before, I can now define an $\mathcal{E}_{T-2\Delta}$ that contains $\mathcal{E}^*$. Proceeding inductively, I can define a nested sequence of sets of sequential equilibria $\mathcal{E}^* \subset \mathcal{E}_0 \subset \ldots \subset \mathcal{E}_{T-\Delta} \subset \mathcal{E}_T$. All equilibria in the set $\mathcal{E}_0$ are belief-free and generate the same belief-free, public continuation payoff process that is described in the proposition. It is easy to show $\mathcal{E}_0$ satisfies the credible threats property, which implies $\mathcal{E}^* = \mathcal{E}_0$. $\square$

Proof of Lemma 1. Suppose $\mathcal{E}_1 \cup \mathcal{E}_2$ does not satisfy the credible threats property. Then there exists an $h_{t-\Delta}$, $(a, m) \in \mathcal{E}_1 \cup \mathcal{E}_2$, $H^P_{t-\Delta}$, and a credible threat $\hat{m}_t(h_{t-\Delta})$ such that P has credible threats opportunity conditional on $h_{t-\Delta}$ and $V_t(h_{t-\Delta})|\hat{m}_t(h_{t-\Delta}) > V_t(H^P_{t-\Delta}, h_{t-\Delta})$.

Without loss of generality, assume $(a, m) \in \mathcal{E}_1$. Then P has a credible threats opportunity given $\mathcal{E}_1$ and conditional on $h_{t-\Delta}$. Given $\mathcal{E}_1$, $\hat{m}_t(h_{t-\Delta})$ continues to be a credible threat. Moreover, the payoffs $V_t(h_{t-\Delta})|\hat{m}_t(h_{t-\Delta})$ and $V_t(H^P_{t-\Delta}, h_{t-\Delta})$ are the same given $\mathcal{E}_1$ and $\mathcal{E}_1 \cup \mathcal{E}_2$. This contradicts the assumption that $\mathcal{E}_1$ satisfies the credible threats property. $\square$

Proof of Theorem 1. Proposition 1 implies there is an obvious correspondence between the portion of a contract after a history $h_{t-\Delta}$ – call it the date t continuation contract given $h_{t-\Delta}$ – and a contract in the version of the model with timeframe $[0, T - t]$.

The proof is by induction on the length of the model timeframe. Fix a Pareto-optimal contract. There is at least some realization of dX_0 such that for all h_0 succeeding $m_0(dX_0)$, the date Δ continuation contract given h_0 is a Pareto-optimal contract in the model with timeframe $[0, T - \Delta]$. Without loss of generality, it is the same Pareto-optimal contract $\mathscr{C}^\Delta$. Now for any realization of dX_0 change the contract so that after P reports $m_0(dX_0)$ the contract randomizes between $\mathscr{C}^\Delta$ and termination using the date 0 public randomizing device. This can be done in a way so that $\mathbf{E}[W_\Delta(h_0) \mid m_0(dX_0)]$ and $\mathbf{E}[V_\Delta(h_0) \mid m_0(dX_0)]$ remain the same. By construction, the altered contract remains incentive-compatible. Relabelling $m_0(dX_0)$ as dX_0 (if two realizations of dX_0 lead to the same $m_0(dX_0)$ then just create two separate messages – it won't affect anything), the contract now has the structure described in Theorem 1 at date 0. By induction, it has the structure described in Theorem 1 at all other dates. $\square$

Proof of Theorem 2. Let $a^*_t(\Delta)$ denote the effort induced by Pareto-optimal contracts at date t. Suppose $\lim_{\Delta \to 0} a^*_t(\Delta) > 0$. Since A is exerting an interior effort, the first-order condition equating marginal cost, $h'(a_t^*(\Delta))\Delta$ to marginal benefit,

$$\left(-\frac{d}{da}\mathbf{P}(dX_t \in \mathit{Bad} \mid a_t^*(\Delta), \Delta)\right) \cdot p_t^*(\Delta) \cdot e^{-r\Delta} S_{t+\Delta}^*(\Delta),$$

must hold. Here, $S_{t+\Delta}^*(\Delta)$ is the (Pareto-optimal) continuation surplus. Since marginal cost $= \Theta(\Delta)$, therefore marginal benefit $= \Theta(\Delta)$. Since $e^{-r\Delta} S_{t+\Delta}^*(\Delta) = \Theta(\Delta^0)$ and, by assumption, $-\frac{d}{da}\mathbf{P}(dX_t \in \mathit{Bad} \mid a_t^*(\Delta), \Delta) = \Theta(\Delta^\alpha)$, therefore $p_t^*(\Delta) = \Theta(\Delta^{1-\alpha})$.

The contribution to surplus of $a_t^*(\Delta)$ relative to zero effort is $= \Theta(\Delta)$. The cost to surplus of $p_t^*(\Delta)$ relative to zero probability of termination is $\mathbf{P}(dX_t \in \mathit{Bad} \mid a_t^*(\Delta), \Delta) \cdot p_t^*(\Delta) = \Theta(\Delta^{\gamma^b + (1-\alpha)})$. Since $\alpha - \gamma^b > 0$, $\Delta^{\gamma^b+(1-\alpha)} \gg \Delta$. Contradiction. □

Proof of Proposition 2. Case 1a: $\gamma_1^g - \gamma_2^b < \alpha_1 - \alpha_2 = 0 < \gamma_1^b - \gamma_2^g$.

It is easy to show $\gamma_1^g = \gamma_2^g = 0$. By the product rule, as $\Delta \to 0$, the derivative of $\mathbf{P}(dX_t = (g_1, b_2) \mid a_t, \Delta)$ with respect to a_t is $A(\Delta) - B(\Delta)$ where $A(\Delta) = \Theta(\Delta^{\alpha_1 + \gamma_2^b})$ and $B(\Delta) = \Theta(\Delta^{\gamma_1^g + \alpha_2})$. Since $\alpha_1 + \gamma_2^b > \gamma_1^g + \alpha_2$, $B(\Delta) \gg A(\Delta)$ and therefore $(g_1, b_2) \in \mathit{Bad}$. By the product rule, as $\Delta \to 0$, the derivative of $\mathbf{P}(dX_t = (b_1, g_2) \mid a_t, \Delta)$ with respect to a_t is $-A(\Delta) + B(\Delta)$ where $A(\Delta) = \Theta(\Delta^{\alpha_1 + \gamma_2^g})$ and $B(\Delta) = \Theta(\Delta^{\gamma_1^b + \alpha_2})$. Since $\alpha_1 + \gamma_2^g < \gamma_1^b + \alpha_2$, $A(\Delta) \gg B(\Delta)$ and therefore $(b_1, g_2) \in \mathit{Bad}$.

Given the results above, $\gamma^b = \min\{\gamma_1^b + \gamma_2^b, \gamma_1^g + \gamma_2^b, \gamma_1^b + \gamma_2^g\} = \min\{\gamma_1^b, \gamma_2^b\}$. $\gamma^g = \gamma_1^g + \gamma_1^g = 0$. $\alpha = \min\{\alpha_1 + \gamma_2^g, \gamma_1^g + \alpha_2\} = \alpha_1 = \alpha_2$.

Case 1b: $\gamma_1^g - \gamma_2^b \le 0 < \alpha_1 - \alpha_2 < \gamma_1^b - \gamma_2^g$.

$\gamma_1^g = 0$, $\gamma_1^b > 0$. $(g_1, b_2) \in \mathit{Bad}$, $(b_1, g_2) \in \mathit{Bad}$. $\gamma^b = \min\{\gamma_1^b + \gamma_2^b, \gamma_1^g + \gamma_2^b, \gamma_1^b + \gamma_2^g\} = \min\{\gamma_1^b + \gamma_2^g, \gamma_2^b\} = \min\{\gamma_1^b, \gamma_2^b\}$. $\gamma^g = \gamma_2^g$. $\alpha = \min\{\alpha_1 + \gamma_2^g, \gamma_1^g + \alpha_2\} = \alpha_2$.

Case 2: $\gamma_1^b - \gamma_2^g \le 0 < \alpha_1 - \alpha_2 < \gamma_1^g - \gamma_2^b$.

$\gamma_1^b = 0$, $\gamma_1^g > 0$. $(g_1, b_2) \in \mathit{Good}$, $(b_1, g_2) \in \mathit{Good}$. $\gamma^b = \gamma_2^b$. $\gamma^g = \min\{\gamma_1^g + \gamma_2^g, \gamma_1^g + \gamma_2^b, \gamma_1^b + \gamma_2^g\} = \min\{\gamma_1^g + \gamma_2^b, \gamma_2^g\} = \min\{\gamma_1^g, \gamma_2^g\}$. $\alpha = \min\{\alpha_1 + \gamma_2^b, \gamma_1^b + \alpha_2\} = \alpha_2$.

Case 3a: $\gamma_1^g - \gamma_2^b \le 0 \le \gamma_1^b - \gamma_2^g < \alpha_1 - \alpha_2$.

$\gamma_1^g = 0$ or $\gamma_2^g = \gamma_1^b = 0$. $(g_1, b_2) \in \mathit{Bad}$, $(b_1, g_2) \in \mathit{Good}$. $\gamma^b = \min\{\gamma_1^b + \gamma_2^b, \gamma_1^g + \gamma_2^b\} = \gamma_2^b$. $\gamma^g = \min\{\gamma_1^g + \gamma_2^g, \gamma_1^b + \gamma_2^g\} = \gamma_2^g$. $\alpha = \min\{\alpha_1 + \gamma_2^b, \gamma_1^b + \alpha_2, \gamma_1^g + \alpha_2\} = \alpha_2$.

Case 3b: $\gamma_1^b - \gamma_2^g \le 0 \le \gamma_1^g - \gamma_2^b < \alpha_1 - \alpha_2$.

$\gamma_1^b = 0$ or $\gamma_1^g = \gamma_2^b = 0$. $(g_1, b_2) \in \mathit{Bad}$, $(b_1, g_2) \in \mathit{Good}$. $\gamma^b = \min\{\gamma_1^b + \gamma_2^b, \gamma_1^g + \gamma_2^b\} = \gamma_2^b$. $\gamma^g = \min\{\gamma_1^g + \gamma_2^g, \gamma_1^b + \gamma_2^g\} = \gamma_2^g$. $\alpha = \min\{\alpha_1 + \gamma_2^b, \gamma_1^b + \alpha_2, \gamma_1^g + \alpha_2\} = \alpha_2$. □

Proof of Lemma 3. Without loss of generality, assume the minimum and maximum values of R are 0 and 1, and assume $s = -\Delta$. Let $\mathcal{F}$ denote the set of all function f

of the form

$$f(X_t) = \begin{cases} 1 & \text{if } X_t - X_s > \rho \\ 0 & \text{if } X_t - X_s \leq \rho \end{cases}$$

for some ρ.

Let $g(X_t, \xi)$ be any function taking values in R that is not completely independent of ξ. Let $f(X_t)$ be the unique function $\in \mathcal{F}$ such that

$$1 - F_t\left(\rho - \sum_{t'=0}^{t} a^g_{[0,t]}(t')\Delta\right) = \mathbf{E}_{a^g_{[0,t]},\xi}\, g(X_t, \xi).$$

Here F_t is the cdf of a normal random variable with mean zero and variance t. Let $a^f_{[0,t]}$ be the largest effort sequence induced by f. Then $a^f_{[0,t]} > a^g_{[0,t]}$.

I now prove there exists a unique $f^* \in \mathcal{F}$ such that the largest effort sequence induced by f^* is strictly larger than any effort sequence induced by any other $f \in \mathcal{F}$. The first part of the proof then implies that $f^* = f^{R,\ t-s}$.

The proof is as follows: Suppose there are two functions f^* and $f \in \mathcal{F}$ such the largest effort sequence induced by f^* equals the largest effort sequence induced by f. I now show that there must be another function $\hat{f} \in \mathcal{F}$ such that largest effort sequence induced by $\hat{f}$ is strictly larger than the largest effort sequence induced by f^* and f:

Let ρ^* and ρ be the thresholds of f^* and f, and let $a^f_{[0,t]}$ be their common largest induced effort sequence. Define $a := \sum_{t'=0}^{t} a^f_{[0,t]}(t')\Delta$. If $a \geq \rho^*$ and $\geq \rho$, then $\rho^* = \rho$, which is a contradiction. So, without loss of generality, assume $a < \rho$. Then the function $\hat{f}$ with threshold $\hat{\rho} = a$ induces a unique effort sequence that is strictly larger than $a^f_{[0,t]}$. □

Proof of Lemma 4. Fix a set $\mathcal{E}$ of sequential equilibria and suppose P has a credible threats opportunity conditional on some h_s. It suffices to show that the credible threat characterized by $f^{R(h_s),\ t-s}$ is time consistent.

Without loss of generality, assume $s = -\Delta$ and the minimum and maximum values of $R(h_s)$ are 0 and 1. Since h_s is empty, I drop all mention of h_s below.

Suppose not. Then there exists a date $t^* < t$ and credible threat m^*_t such that $V_{t^*+\Delta}|\{m^*_t, a_{[s,t^*]}|m_t\} > V_{t^*+\Delta}|m_t$. The proof of Lemma 3 implies without loss of generality, m^*_t viewed as a function of X_t is an element of $\mathcal{F}$ with some threshold ρ^*.

I now show that there is a credible threat that induces a higher effort sequence than m_t which is a contradiction. Let $a^*_{[0,t]}$ be the largest effort sequence induced by m^*_t. Define F_t to be the cdf of a normal random variable with mean zero and variance

t. Create the equation

$$\eta(x) := 1 - F_t \left(\rho^* - \sum_{t'=0}^{t} a^*_{[0,t]}(t')\Delta - x \right)$$

η is an increasing logistic-shaped function with a convex lower half and a concave upper half. Since $a^*_{[0,t]}$ is a best-response to ρ^*, it must be that

$$\eta'(0) = e^{r(t-t')} h'(a^*_{[0,t]}(t')) \quad \forall t' \in [0,t] \tag{1}$$

The proof of Lemma 3 implies it is without loss of generality to assume 0 is in the domain of the concave upper half of $\eta(x)$.

The first-order conditions for effort imply that either $a_{[0,t]} \leq a^*_{[0,t]}$ or $a_{[0,t]} > a^*_{[0,t]}$. First suppose $a_{[0,t]} > a^*_{[0,t]}$. Now consider the scenario where the worker chooses to follow a up to date t^* but then switches to $a^*_{[0,t]}$ starting from date $t^* + \Delta$. At date $t^* + \Delta$, the marginal benefit of the date t' effort at level $a^*_{[0,t]}$ is $e^{-r(t-t^*)}\eta'(\sum_{k=0}^{t^*}(a_{[0,t]}(k) - a^*_{[0,t]}(k))\Delta)\Delta$ for any $t' \in [t^* + \Delta, t]$. The marginal cost is $e^{-r(t'-t^*)}h'(a^*_{[0,t]}(t'))\Delta$.

Since $\sum_{k=0}^{t^*}(a_{[0,t]}(k) - a^*_{[0,t]}(k))\Delta > 0$ and $\eta(0)$ is in the concave upper half, it must be that $\eta'(\sum_{k=0}^{t^*}(a_{[0,t]}(k) - a^*_{[0,t]}(k))\Delta) < \eta'(0)$. Combining this with (1) implies that under the proposed scenario, starting at date t^*, the marginal benefit of effort going forward is less than the marginal cost. This means that $a_{[t^*+\Delta,t]}|\{m^*_t, a_{[0,t^*]}|m_t\}$ must be smaller than $a^*_{[0,t]}$ on the interval $[t^* + \Delta, t]$. By assumption, this implies that $a_{[t^*+\Delta,t]}|\{m^*_t, a_{[0,t^*]}|m_t\} < a^*_{[0,t]} < a_{[0,t]}$ on the interval $[t^* + \Delta, t]$. Contradiction. So $a^*_{[0,t]} \geq a_{[0,t]}$. But $a^*_{[0,t]} \neq a_{[0,t]}$, so $a^*_{[0,t]} > a_{[0,t]}$. $\square$

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